DE LA RECHERCHE À L'INDUSTRIE

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A 2D sliding algorithm for Eulerian multimaterial simulations

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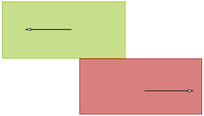
Introduction

Context

- Multimaterial simulations of compressible hydrodynamics phenomena
- Eulerian method: fixed grid
- Time splitting:
  - Lagrangian phase: predictor / corrector scheme with nodal velocities
  - advection phase: Alternating Directions method
- Eulerian methods : materials are welded compared to
  - ALE methods [DEL PINO and LABOURASSE, submitted]
  - Lagrangian methods [WILKINS, 99], [CARAMANA, 09], [KUCHARIK, LISKA, BEDNARIK and LOUBÈRE, 11], [DEL PINO and LABOURASSE, submitted], [CLAIR, DESPRÉS and LABOURASSE, 12]



  which treat sliding in a more natural way
- Material nodal velocities

Outline

- Sliding method
  - General description
  - Lagrangian phase
  - Remapping phase
- Numerical results
  - Without vacuum around sliding materials
  - With vacuum around sliding materials

Outline

- Sliding method
  - General description
  - Lagrangian phase
  - Remapping phase

Numerical results

  Without vacuum around sliding materials

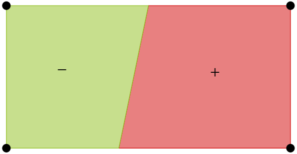
  With vacuum around sliding materials

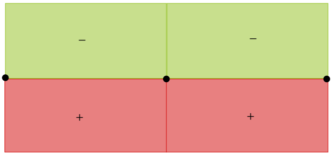
General description

In this presentation we limit ourselves to only two sliding materials, labeled by $+$ and $-$.

Definition

We call *mixed node* a node of a mixed cell (*i.e.* a cell containing more than one material) or shared by pure cells containing different materials.

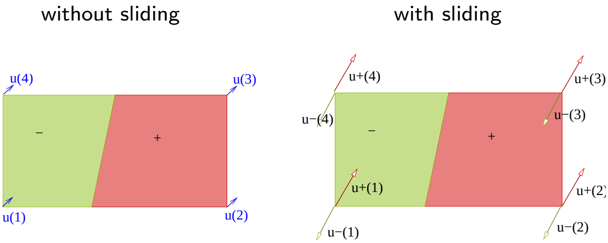




Sliding method

Data structure for sliding

- For each mixed node i, there exist two material nodal velocities denoted by $\mathbf{u}_+(i)$ and $\mathbf{u}_-(i)$.



- For each pure cell k, we define by δ the nature of the material ($+$ or $-$), $V(k)$ the volume, $\rho(k)$ the density and $p(k)$ the pressure of the cell.
- For each mixed cell k, we define by $V_\delta(k)$ the partial volume, $\rho_\delta(k)$ the partial density and $p_\delta(k)$ the partial pressure of material $\delta \in \{-, +\}$ in k.

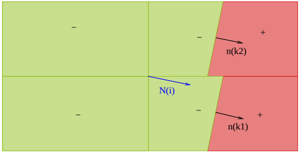
Sliding method

Data structure for sliding

- Using partial volumes $V_\delta(k)$ of material δ, we define in each cell k (pure or mixed), two volumic fractions:

$$f_\delta(k) = \frac{V_\delta(k)}{V_-(k) + V_+(k)}, \quad \delta \in \{-,+\}, \quad \text{so that: } f_-(k) + f_+(k) = 1$$

- For each mixed node i , we define by $\mathbf{N}(i)$ the unit normal as:



$$\mathbf{N}(i) = \frac{\mathbf{n}(k_1) + \cdots + \mathbf{n}(k_p)}{||\mathbf{n}(k_1) + \cdots + \mathbf{n}(k_p)||}$$

where $\mathbf{n}(k)$ are the unit normals at interface between $+$ and $-$ in mixed cells

in 2D structured grid, $p \leq 4$

Sliding method

Lagrangian phase

Momentum conservation is satisfied by solving for each mixed node:

$$\begin{cases} \rho_- \dfrac{d\mathbf{u}_-}{dt} + \nabla p_- = f_+ \mathbf{S} \\ \rho_+ \dfrac{d\mathbf{u}_+}{dt} + \nabla p_+ = -f_- \mathbf{S} \end{cases} \quad (1)$$

where, for all mixed point i and for all mixed cell k:

- $\mathbf{S}(i) = -\frac{1}{\varepsilon} \left| \mathbf{u}_-(i) \cdot \mathbf{N}(i) - \mathbf{u}_+(i) \cdot \mathbf{N}(i) \right| \left(\mathbf{u}_-(i) \cdot \mathbf{N}(i) - \mathbf{u}_+(i) \cdot \mathbf{N}(i) \right) \mathbf{N}(i)$

  is a relaxation term which ensures the condition of non penetration of materials

$$\mathbf{u}_- \cdot \mathbf{N} = \mathbf{u}_+ \cdot \mathbf{N}$$

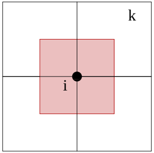
- parameter ε has the dimension of a length divided by a density

$$\varepsilon \equiv \frac{\varepsilon_0}{2} \Delta x \left(\frac{1}{\rho_-} + \frac{1}{\rho_+} \right), \quad \text{with} \quad \varepsilon_0 \approx 10^{-4}, \text{and } \Delta x \text{ is cell length}$$

Sliding method

Lagrangian phase

- $$\rho_\delta(i) = \frac{\sum_{k\in i} V_\delta(k)\rho_\delta(k)}{\sum_{k\in i} V_\delta(k)} = \frac{\text{Mass of } \delta \text{ in nodal cell}}{\text{Volume of nodal cell}}$$



- $$f_\delta(i) = \frac{\sum_{k\in i} V_\delta(k)f_\delta(k)}{\sum_{k\in i} V_\delta(k)} = \frac{\text{Volume of } \delta \text{ in nodal cell}}{\text{Volume of nodal cell}}$$

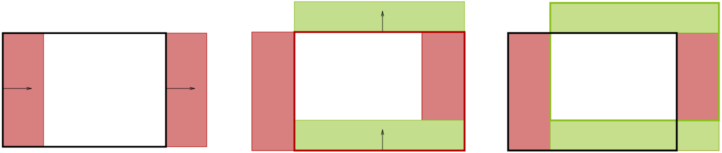
- We solve system (1) in two steps:

$$\begin{cases} \rho_- \dfrac{d\mathbf{u}_-}{dt} + \nabla p_- = 0 \\ \rho_+ \dfrac{d\mathbf{u}_+}{dt} + \nabla p_+ = 0 \end{cases} \quad \text{and} \quad \begin{cases} \rho_- \dfrac{d\mathbf{u}_-}{dt} = f_+ \mathbf{S} \\ \rho_+ \dfrac{d\mathbf{u}_+}{dt} = -f_- \mathbf{S} \end{cases}$$

- We use separate coordinates for each material δ, then nodes i evolve first with the material velocities $\mathbf{u}_+(i)$ and second with $\mathbf{u}_-(i)$.

Sliding method

Remapping phase: Alternating Directions method

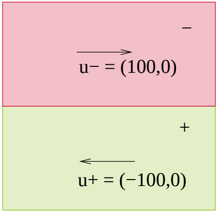


- Computation of an edge velocity $\mathbf{u}_{e,\delta}(k)$ using $\mathbf{u}_{\delta}(i)$ (mean of $\mathbf{u}_{\delta}(i)$), for all cell k
- Remapping of all quantities of material $\delta \in \{-,+\}$ with $\mathbf{u}_{e,\delta}$
- Algorithm:
  - remapping in the first direction
    - remapping of all physical quantities "$+$" with $\mathbf{u}_{e,+}$
    - remapping of all physical quantities "$-$" with $\mathbf{u}_{e,-}$
  - remapping in the second direction
    - remapping of all physical quantities "$+$" with $\mathbf{u}_{e,+}$
    - remapping of all physical quantities "$-$" with $\mathbf{u}_{e,-}$

Outline

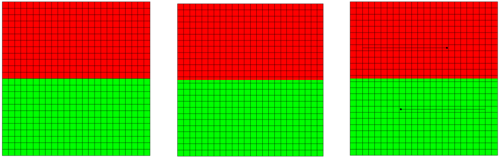
Without vacuum: first test case

Initialization

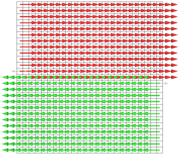


$p = 0,\ \rho = 1,\ dt = 10^{-6}$
for $\delta \in \{-, +\}$

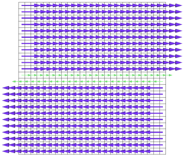
Initial grids and Lagrangian particles after 1000 it



After 1000 iterations, comparison:



with



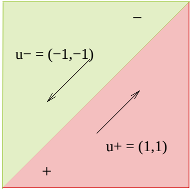
without (boundary layer)

sliding algorithm

Numerical results

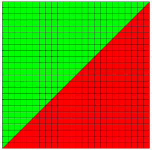
Without vacuum: second test case

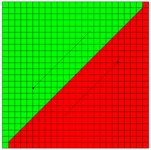
Initialization



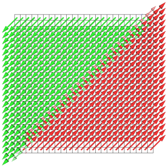
$p = 0,\ \rho = 1,\ dt = 10^{-6}$
for $\delta \in \{-,+\}$

Initial grid and Lagrangian particles after 1000 it





After 1000 iterations, comparison:



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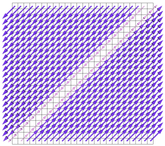


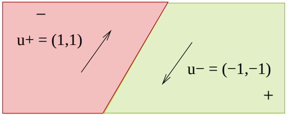
without
(boundary layer)

sliding algorithm

Numerical results

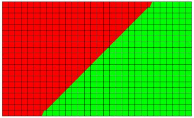
Without vacuum: third test case

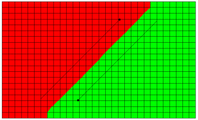
Initialization



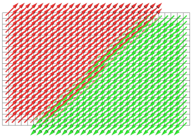
$p = 0,\ \rho = 1,\ dt = 10^{-6}$
for $\delta \in \{-, +\}$

Initial grid and Lagrangian particles after 1000 it

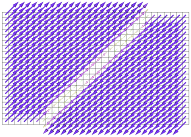




After 1000 iterations, comparison:



with

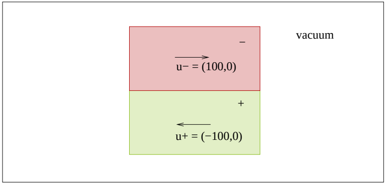


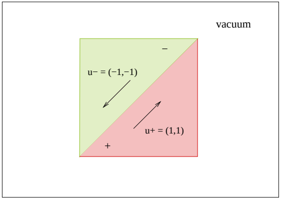
without
(boundary layer)

sliding algorithm

With vacuum around sliding materials

Examples





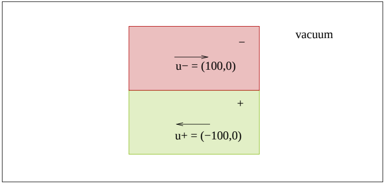
Vacuum treatment during remapping phase:

- for each step of remapping phase (with $\mathbf{u}_{e,+}$ or $\mathbf{u}_{e,-}$), for each direction, remapping of volume for vacuum to ensure for every cell k: $\sum_{\delta} f_{\delta}(k) = 1$, where $\delta \in \{+, -, \text{vacuum}\}$

Numerical results

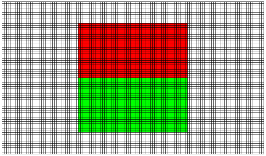
With vacuum: first test case

Initialization

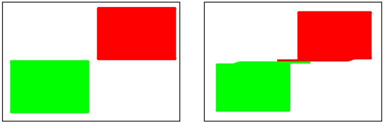


$p = 0,\ \rho = 1,\ dt = 10^{-6}$
for $\delta \in \{-,+\}$

Initial grid and definition of the materials



After 5500 iterations, comparison:

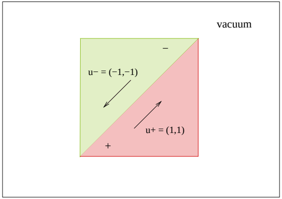


with without
sliding algorithm

Numerical results

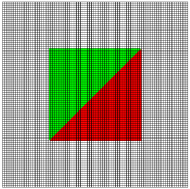
With vacuum: second test case

Initialization

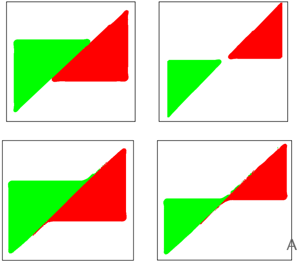


$p = 0,\ \rho = 1,\ dt = 10^{-6}$
for $\delta \in \{-, +\}$

Initial grid and definition of the materials



After 310 (l) and 720 (r) iterations, comparison:



with

sliding
algorithm

without

Conclusion and perspectives

Conclusion

- new 2D Eulerian sliding algorithm has been implemented
- no problem to handle mixed cells
- special treatment for vacuum in remapping phase
- several test cases which validate our approach

Perspectives

- N sliding materials ($N > 2$)
- extension to 3D