High Dimensional Switched Systems: Control and Observation

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Introduction

Framework

- Goal: control the evolution of an operating system with the help of actuators and sensors
- Framework of the switched control systems: one selects the working modes of the system over time, every mode is described by differential equations (ODEs or PDEs)
- Application to medium/high dimensional systems:
  - Model Order Reduction
  - Error bounding
  - State space bisection

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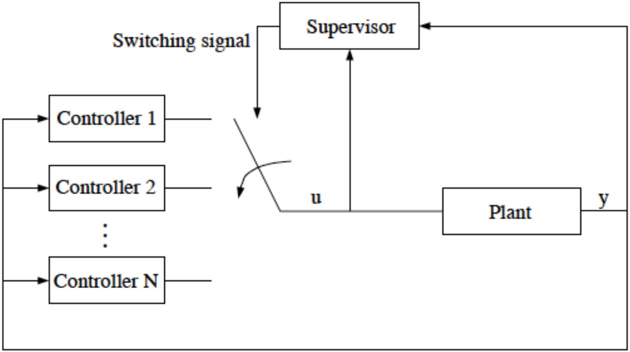
We focus here on sampled switched systems: switching instants occur periodically every τ ($\rightsquigarrow \sigma$ is constant on $[i\tau, (i+1)\tau)$)

Controlled Switched Systems: Schematic View



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NB: classic stabilization impossible here (no common equilibrium pt)
$\rightsquigarrow$ practical stability

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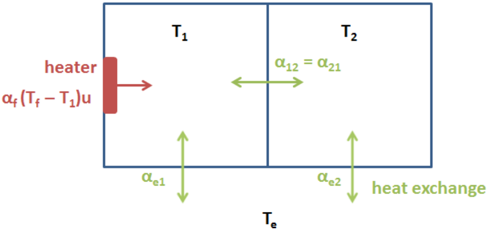

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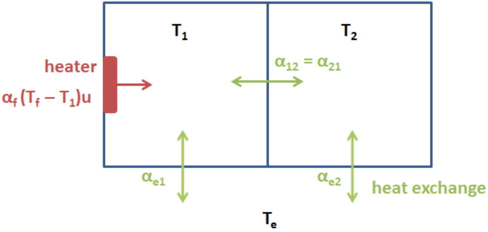

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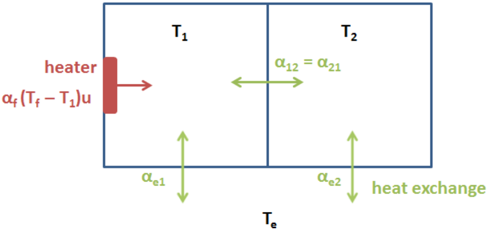

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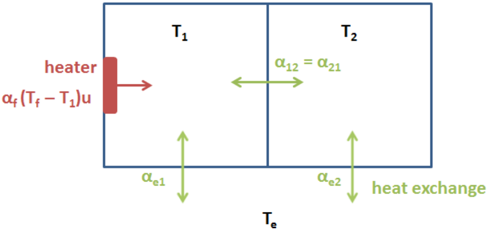

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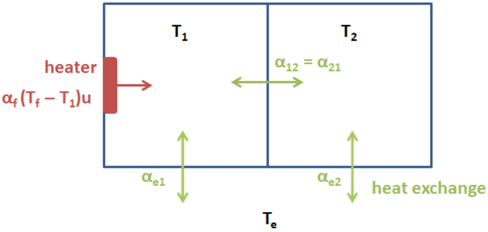

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NB: Each mode has its basic proper equilibrium point; by appropriate switching, one can drive the system to a specific stability zone

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- Example of stability property to be checked: temperature regulation

$$|T_i(t) - T_{reference}| \leq \varepsilon \text{ as } t \to \infty$$

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2 State Space Decomposition

3 Control of high dimensional switched systems

4 Observation of high dimensional switched systems

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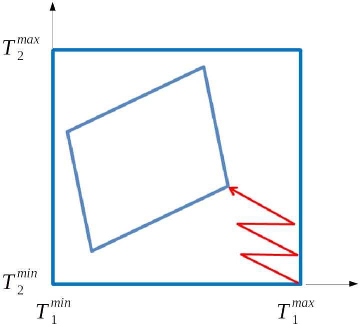
- Look for a *pattern* which maps R into R



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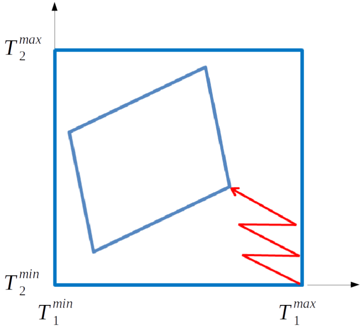

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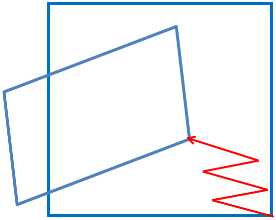

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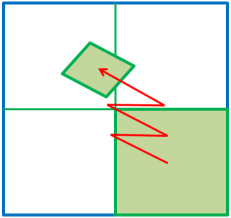

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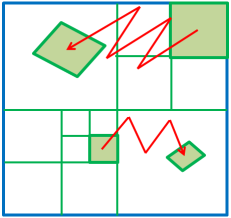

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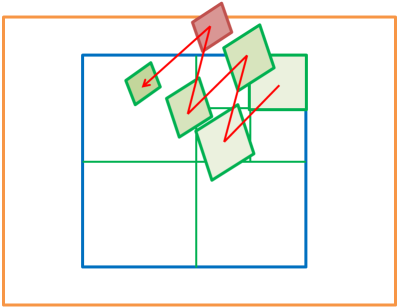
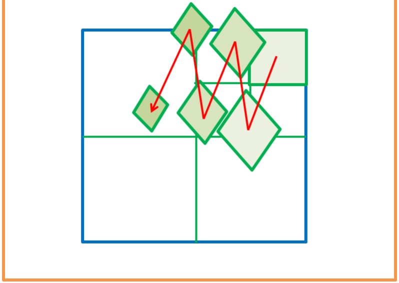
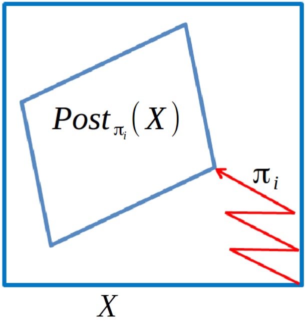
- Extension for safety: the unfolding must stay in the safety set S.





Post Set Operators



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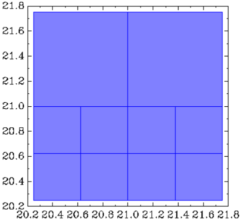

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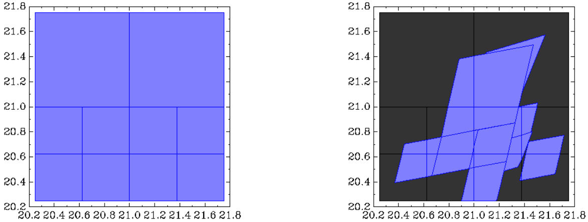

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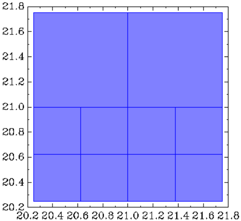

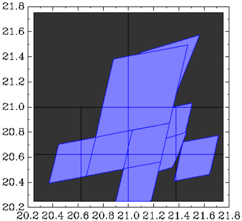

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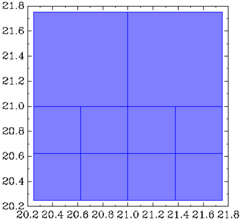

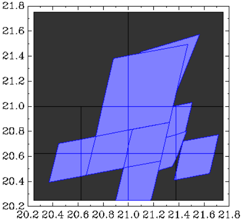

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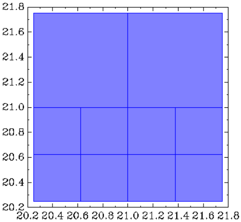

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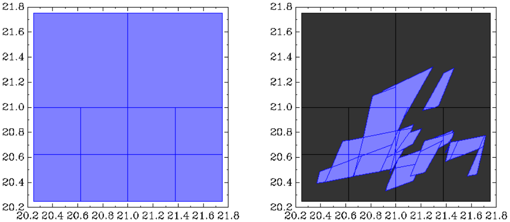

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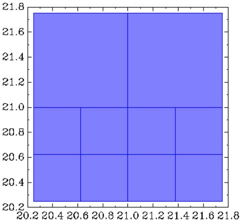

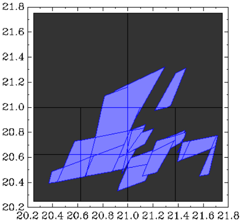

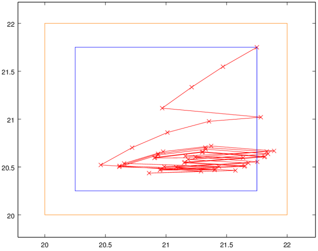

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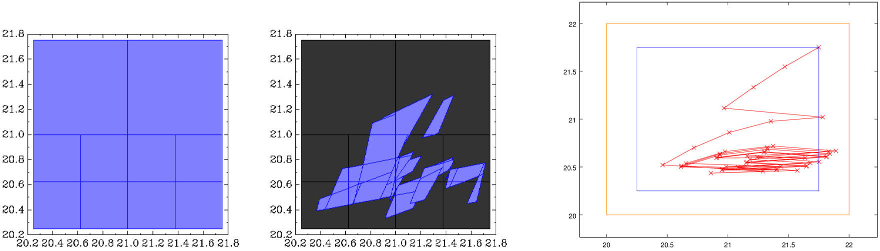

Figure : Decomposition (left) ; unfolding (middle) ; unfolded trajectory (right) in plane (T_1, T_2)

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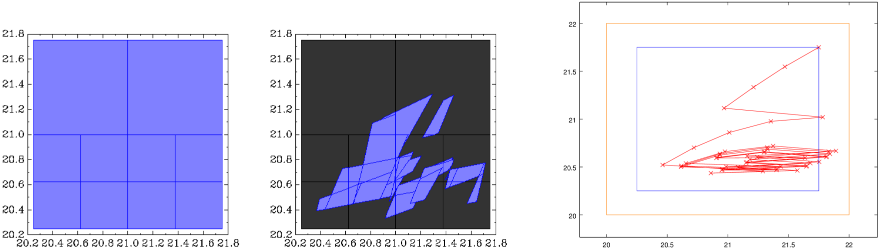

Figure : Decomposition (left) ; unfolding (middle) ; unfolded trajectory (right) in plane (T_1, T_2)

Decomposition found for $k = 4, d = 3$.

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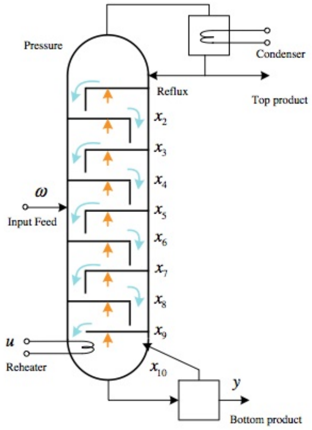
A Sampled Switched System with Output

A distillation column



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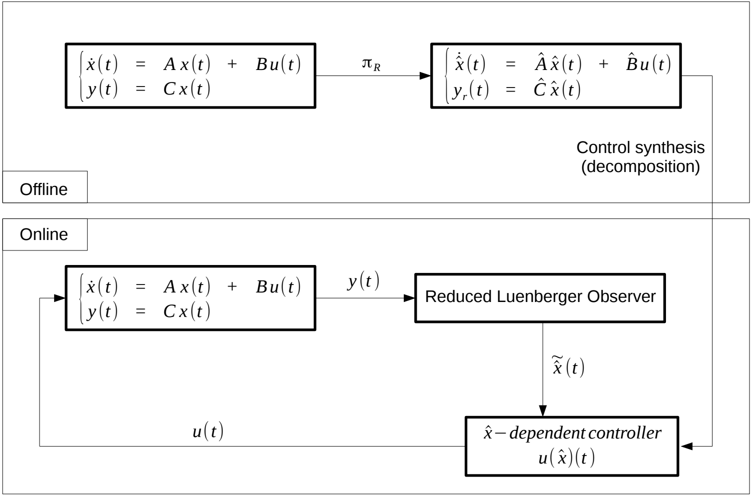
Computational cost of decomposition: at most in $O(2^{nd}N^k)$.

Dealing with high dimensionality : model reduction



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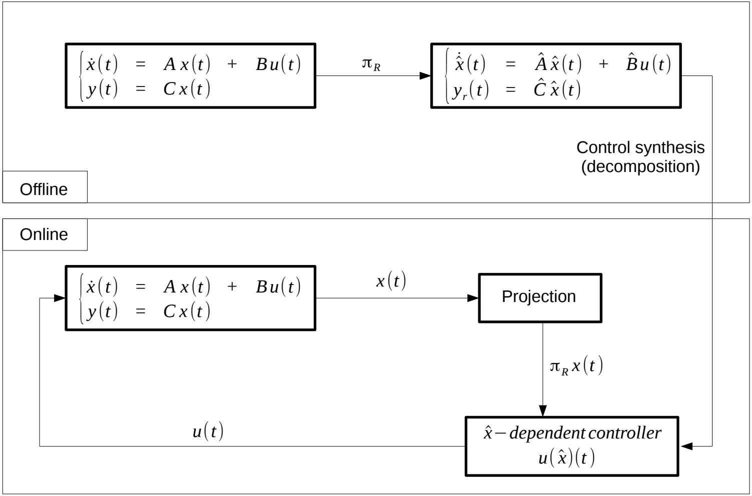

Outline

1. Switched Systems
2. State Space Decomposition
3. Control of high dimensional switched systems
   - Model Order Reduction
   - Guaranteed offline control
   - Guaranteed online control
4. Observation of high dimensional switched systems

Outline

Model Order Reduction by Projection

Construction of a reduced order system $\hat{\Sigma}$ of order $n_r < n$:

$$\hat{\Sigma} : \begin{cases} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}u(t), \\ y_r(t) &= \hat{C}\hat{x}(t). \end{cases}$$

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Output and state trajectory error [2]

After application of a pattern of length j

- the error between y and y_r is bounded by:

$$\varepsilon_y^j = \|u(\cdot)\|_\infty^{[0,j\tau]} \int_0^{j\tau} \| \begin{bmatrix} C & -\hat{C} \end{bmatrix} \begin{bmatrix} e^{tA} & \\ & e^{t\hat{A}} \end{bmatrix} \begin{bmatrix} B \\ \hat{B} \end{bmatrix} \| dt \quad + \sup_{x_0 \in R_x} \| \begin{bmatrix} C & -\hat{C} \end{bmatrix} \begin{bmatrix} e^{j\tau A} & \\ & e^{j\tau\hat{A}} \end{bmatrix} \begin{bmatrix} x_0 \\ \pi_R x_0 \end{bmatrix} \|.$$

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Reduced Order Control

Two systems:

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$\Rightarrow$ reduced-order control
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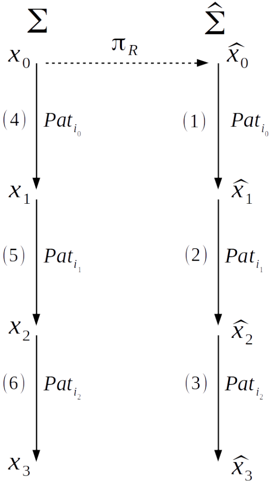
Questions:

- How is it applied?
- Is the reduced-order control effective at the full-order level?

Outline

Offline Procedure



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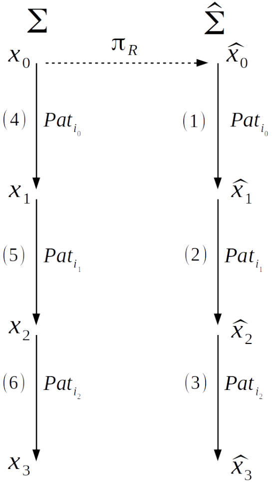
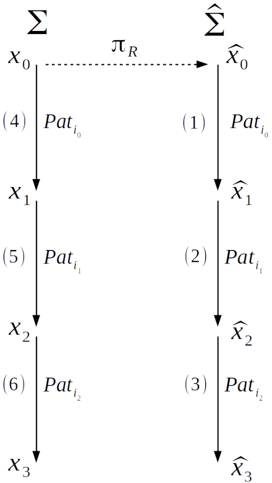


1. Projection of the initial state x_0

Offline Procedure



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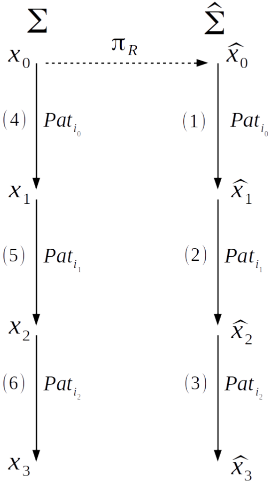

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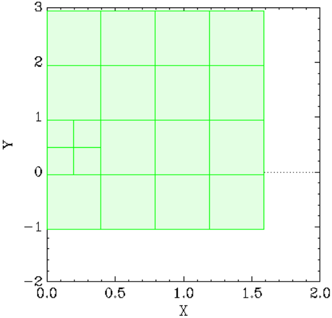
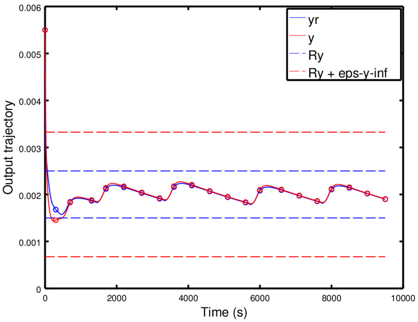
Consequence: the output of the full order system is sent in $R_y + \varepsilon_y^\infty$.

Guaranteed Offline Control

Simulation on a linearized model of a distillation column: $n = 11$ and $n_r = 2$:





Outline

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2 State Space Decomposition
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   - Model Order Reduction
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   - Guaranteed online control
4 Observation of high dimensional switched systems
   - Observation of switched systems
   - Numerical test of a reduced order observer

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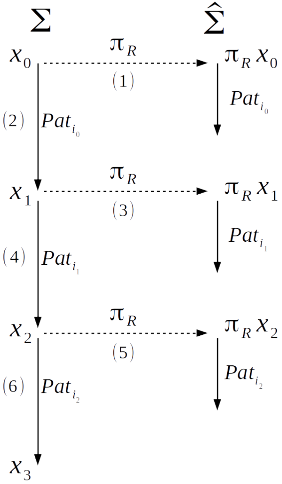

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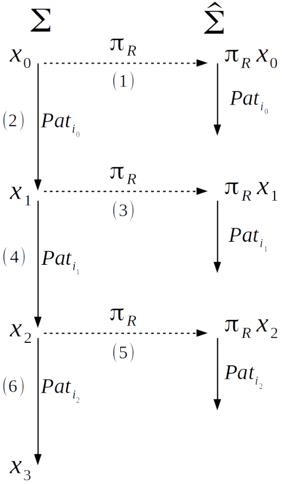

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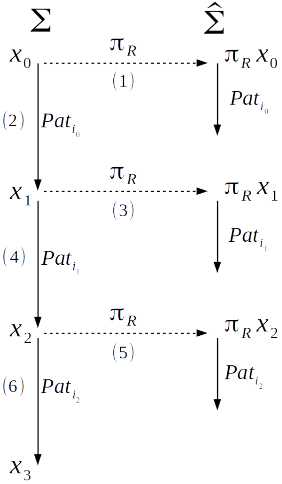

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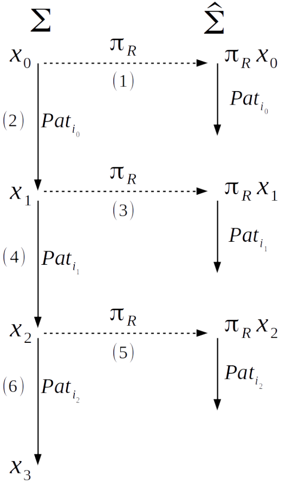
Online Procedure



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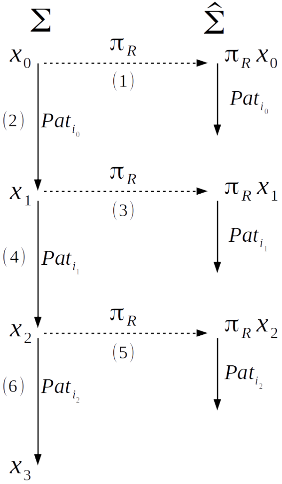

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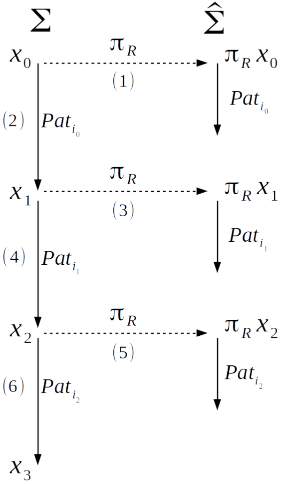

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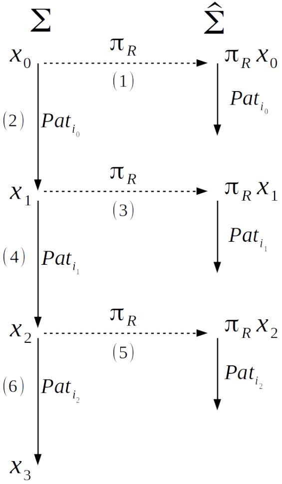

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Solution: Compute an ε-decomposition

definition

A ε-decomposition Δ of R_x is a set of couples $\{(V_i, Pat_i)\}_{i \in I}$ such that:

- $\bigcup_{i \in I} V_i = R_x$
- $\forall i \in I \ Post_{Pat_i}(V_i) \subseteq R_x - \varepsilon_x^{|Pat_i|}$
- $\forall i \in I \ Post_{Pat_i,C}(V_i) \subseteq R_y$ (y-convergence)

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- thus, at every step k:

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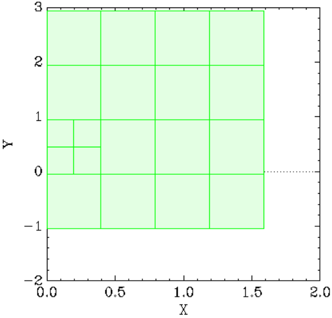
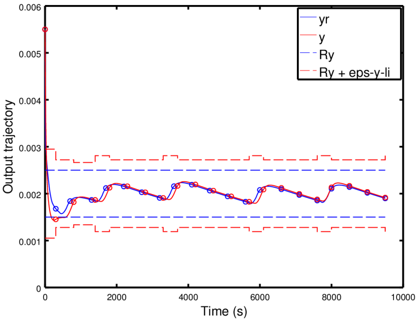
Guaranteed Online Control

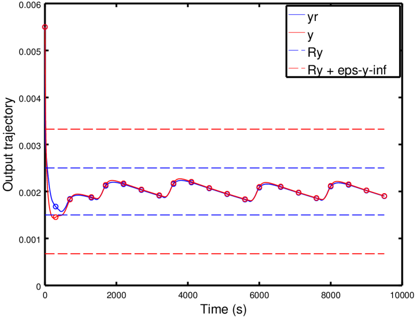
Simulation on a linearized model of a distillation column: $n = 11$ and $n_r = 2$:

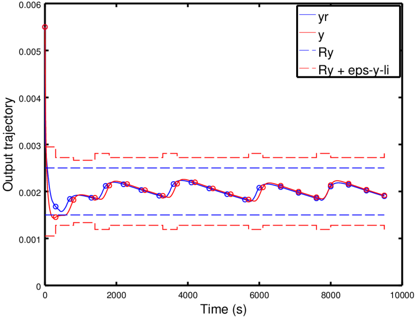




Remark: Output trajectory error depending on the length of the applied pattern: much lower than the infinite bound ε_y^∞

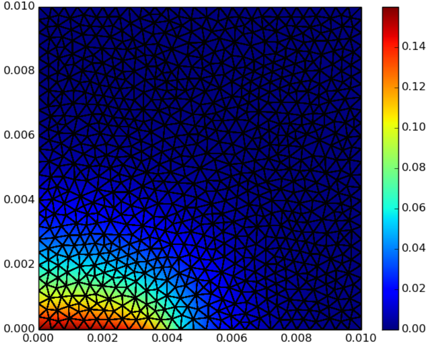
Comparison of the Two Procedures





Other Applications

- Control of the temperature of a square plate discretized by finite elements: offline and online control
  $n = 897$



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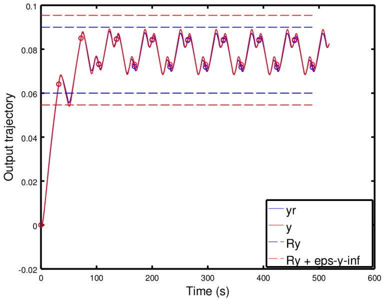

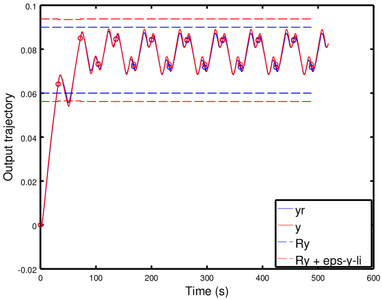

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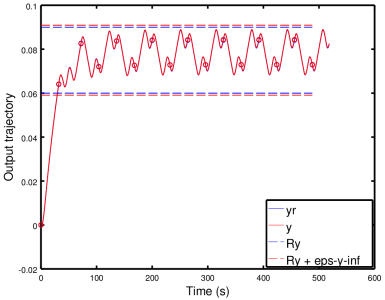

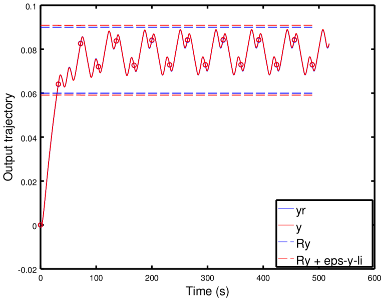

Other Applications

- Vibration (online) control of a cantilever beam:
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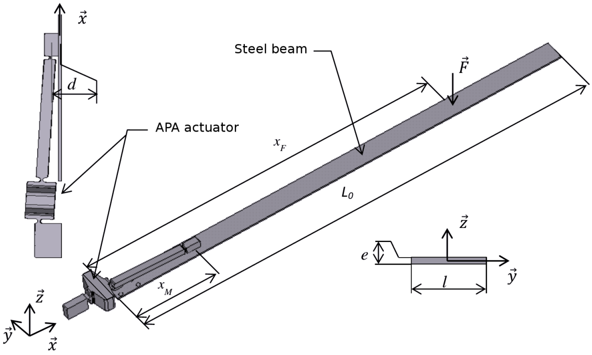

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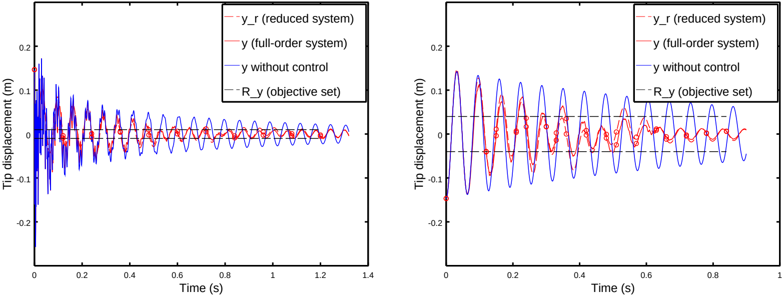

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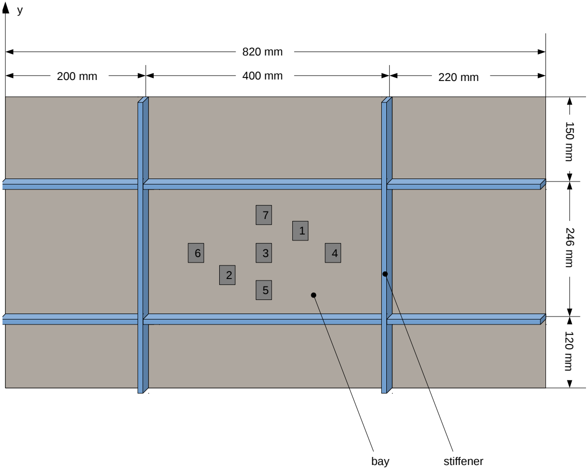

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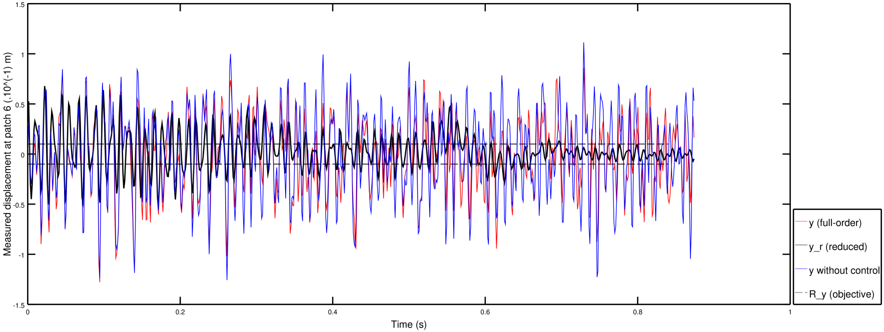

Outline

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Partial observation (without model reduction)

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Given the switched system:

$$\Sigma : \begin{cases} \dot{x}(t) & = Ax(t) + Bu(t), \\ y(t) & = Cx(t). \end{cases}$$

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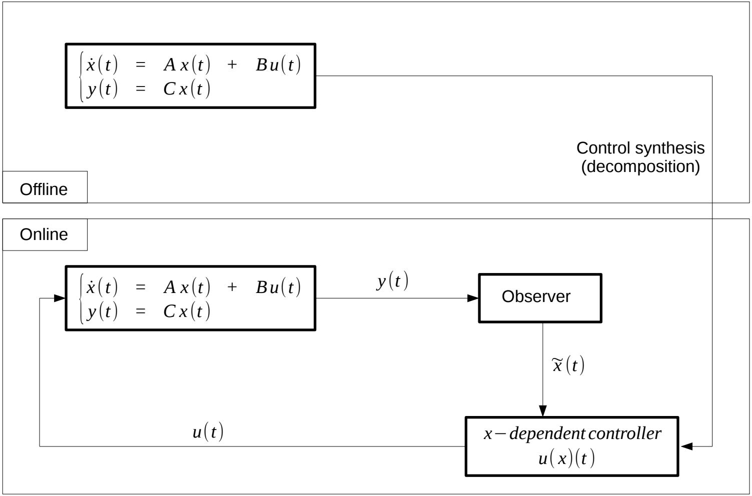

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$\Rightarrow$ Kalman filter, High gain observer, Luenberger observer?

Why the Luenberger observer?

- Dynamics of the Luenberger observer:

$$\dot{\tilde{x}} = A\tilde{x} - L(u)(C\tilde{x} - y) + Bu, \quad L(u) \in \mathbb{R}^{n \times m}$$

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- Objective: find a strategy such that the observer converges: $\eta(t) = |\tilde{x}(t) - x(t)| \underset{t\to+\infty}{\longrightarrow} 0$

Properties of the Luenberger observer

Hypotheses:

- $\exists P > 0, \quad s.t. \quad P(A + L(u)C) + (A + L(u)C)^\top P \leq 0 \quad \forall u.$
- (Dwell-time: $\tau > 0$)

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[Serres, Vivalda, Riedinger, IEEE Trans.Auto.Cont. 2011]

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Proof based on the study of

$$\dot{e} = (A - L(u)C)e$$

where $e(t) = x(t) - \tilde{x}(t)$

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Observer based decomposition

Supposing that the initial reconstruction error is inferior to η_0

definition

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$$Post_{\tilde{\Delta}}(R_x + \eta_0) \subseteq R_x - \eta_0 \quad \text{and} \quad Post_{\Delta,C}(R_x + \eta_0) \subseteq R_y.$$

Outline

Numerical implementation with model reduction

An ε-decomposition is performed.

Use of a reduced Luenberger observer:

$$\dot{\tilde{\hat{x}}} = \hat{A}\tilde{\hat{x}} - L(u)(\hat{C}\tilde{\hat{x}} - Cx) + \hat{B}u, \quad L(u) \in \mathbb{R}^{n_r \times m}$$

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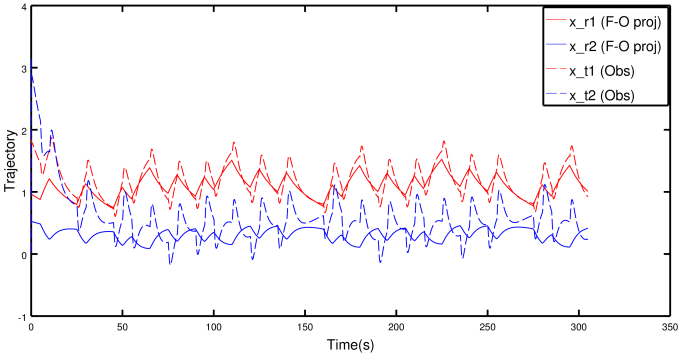
Simulation on the thermal plate problem:

Full-order system initialized at 0.06^{897}, observer initialized at 0^{897}



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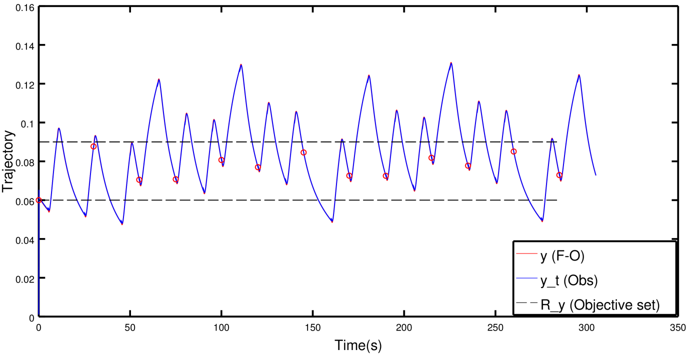

Conclusions

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Future work

- Decomposition using dimensionality reduction (projection on more adapted reduced spaces using post-process techniques)
- Improvement of model reduction techniques (adapted to hyperbolic and non-linear systems)
- Control of non-linear systems/PDEs

Some References

Laurent Fribourg, Ulrich Kühne, and Romain Soulat.
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Thank you ! Questions?