Du dual de BV à la détection d'empreintes digitales

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sur un modèle mathématique de

généralisant un fameux modèle de :

avec des expériences utilisant l'article en ligne de :

basées sur une analyse des modèles due à :

sur un modèle proposé par :

et par un algorithme d'optimisation inventé par :

(1) Yves Meyer *Oscillating patterns in image processing and nonlinear evolution equations (*2001)

(2) L. I. Rudin, S. Osher, and E. Fatemi *Nonlinear total variation based noise removal algorithms* (1992)

(3) V. Le Guen *Cartoon + Texture Image Decomposition by the TV-L1 Model*, (2014)

(4) J.-F. Aujol, G. Gilboa, T. Chan & S. Osher, *Structure-Texture Image Decomposition* (2005)

(5) M. Nikolova *Minimizers of cost-functions involving non-smooth data-fidelity terms….*

(6) A. Chambolle and T. Pock, *A first-order primal-dual algorithm for convex problems …*(2011)

Généalogie des fonctions à variation bornée (BV):

https://en.wikipedia.org/wiki/Bounded_variation

According to Boris Golubov, *BV* functions of a single variable were first introduced by Camille Jordan, in the paper (Jordan 1881) dealing with the convergence of Fourier series. The first successful step in the generalization of this concept to functions of several variables was due to Leonida Tonelli,[1] who introduced a class of *continuous* *BV* functions in 1926 (Cesari 1986, pp. 47–48), to extend his direct method for finding solutions to problems in the calculus of variations in more than one variable. Ten years after, in (Cesari 1936), Lamberto Cesari *changed the continuity requirement* in Tonelli's definition *to a less restrictive integrability requirement*, obtaining for the first time the class of functions of bounded variation of several variables in its full generality: as Jordan did before him, he applied the concept to resolve of a problem concerning the convergence of Fourier series, but for functions of *two variables*. After him, several authors applied *BV* functions to study Fourier series in several variables, geometric measure theory, calculus of variations, and mathematical physics.

Renato Caccioppoli and Ennio de Giorgi used them to define measure of nonsmooth boundaries of sets (see the entry "*Caccioppoli set*" for further information). Olga Arsenievna Oleinik introduced her view of generalized solutions for nonlinear partial differential equations as functions from the space *BV* in the paper (Oleinik 1957), and was able to construct a generalized solution of bounded variation of a first order partial differential equation in the paper (Oleinik 1959): few years later, Edward D. Conway and Joel A. Smoller applied *BV*-functions to the study of a single nonlinear hyperbolic partial differential equation of first order in the paper (Conway & Smoller 1966), proving that the solution of the Cauchy problem for such equations is a function of bounded variation, provided the initial value belongs to the same class. Aizik Isaakovich Vol'pert developed extensively a calculus for *BV* functions: in the paper (Vol'pert 1967) he proved the chain rule for BV functions and in the book (Hudjaev & Vol'pert 1985) he, jointly with his pupil Sergei Ivanovich Hudjaev, explored extensively the properties of *BV* functions and their application. His chain rule formula was later extended by Luigi Ambrosio and Gianni Dal Maso in the paper (Ambrosio & Dal Maso 1990).

Les trois formules fondamentales pour decomposer une image f = cartoon u + texture v

$TV\text{-}L^2$ **(ROF)**

$$\inf_{(u,v)\in BV\times L^2/f=u+v}\left(\int |Du| + \lambda\|v\|_{L^2}^2\right) \qquad (1)$$

$TV\text{-}G$ **(Meyer)**

$$\inf_{(u,v)\in BV\times G/f=u+v}\left(\int |Du| + \lambda\|v\|_{G}\right) \qquad (2)$$

$TV\text{-}L^1$ (Nikolova, Aujol)

$$\inf_{(u,v)\in BV\times L^1/f=u+v}\left(\int |Du| + \lambda\|v\|_{L^1}\right) \qquad (5)$$

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Image f =

Archive de IPOL : http://demo.ipol.im/demo/103/archive

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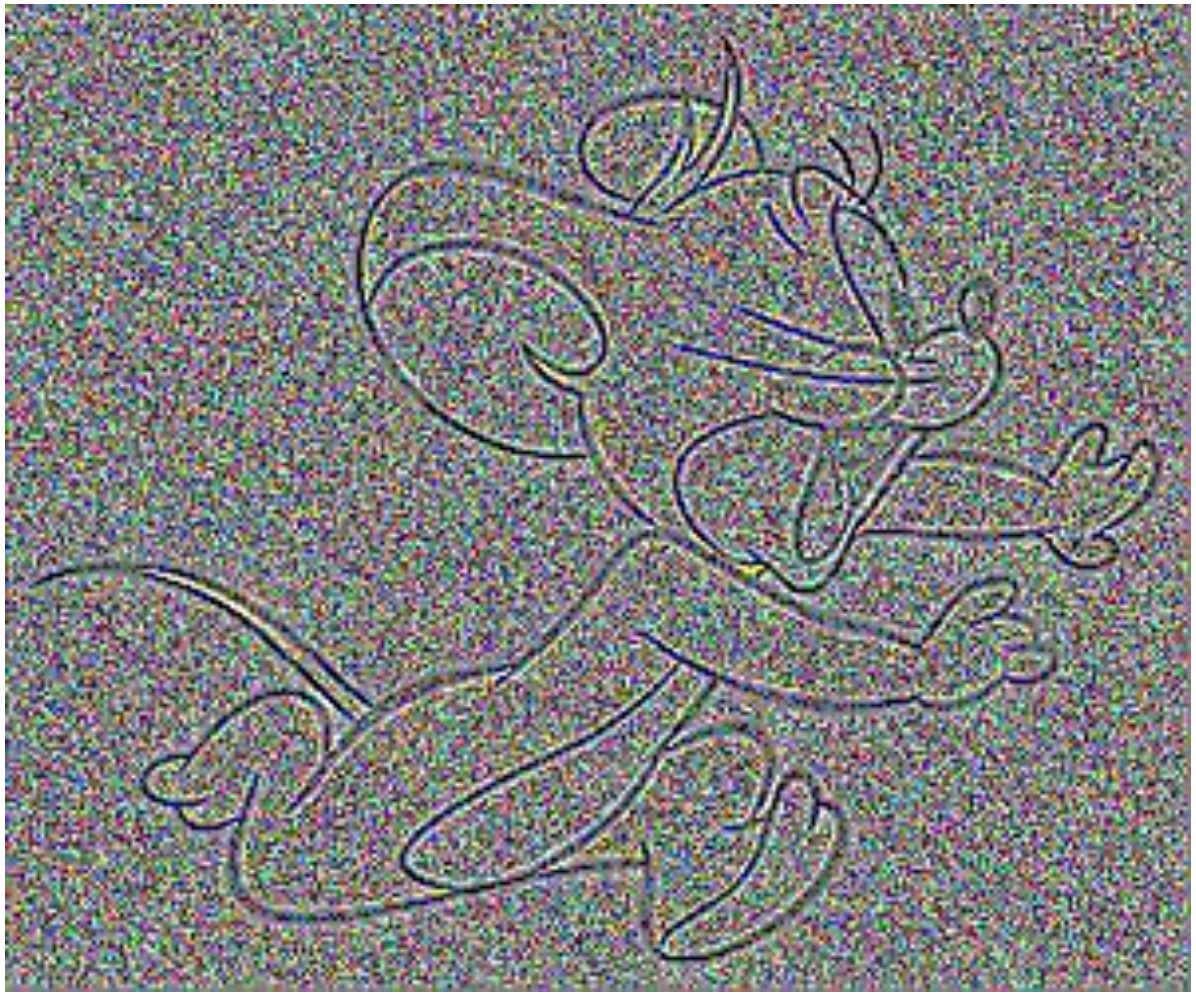
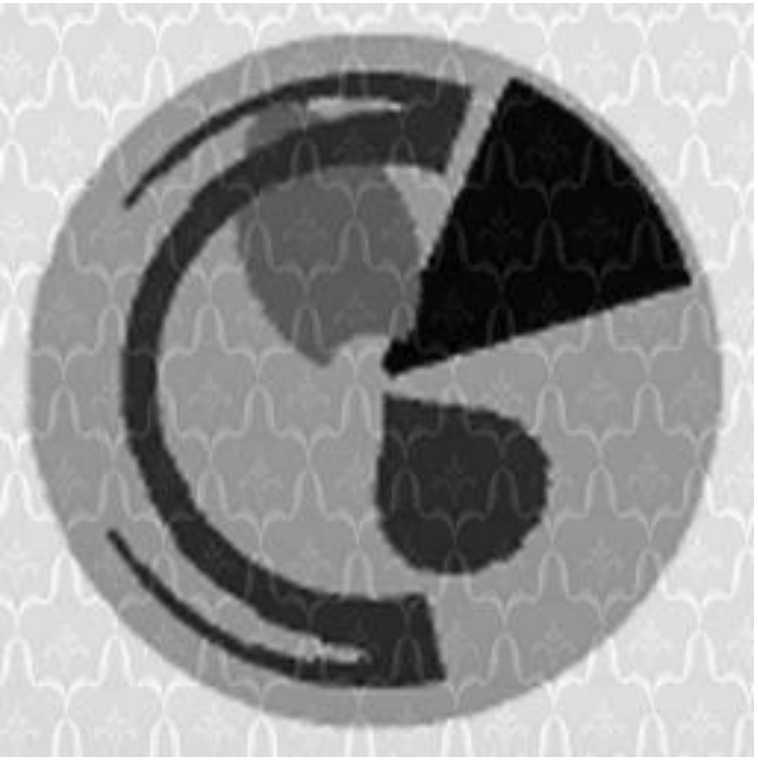
Cartoon u (à variation bornée) +

Archive de IPOL : http://demo.ipol.im/demo/103/archive



Texture v (très oscillante)

Archive de IPOL : http://demo.ipol.im/demo/103/archive



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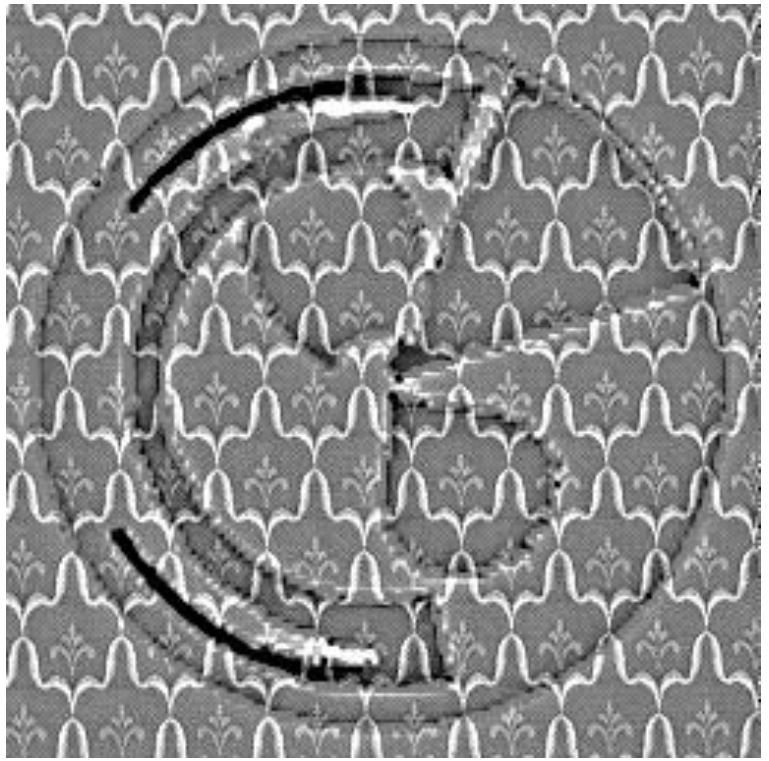

Image f = Cartoon u + Texture v

Archive de IPOL : http://demo.ipol.im/demo/103/archive

Archive de IPOL : http://demo.ipol.im/demo/103/archive

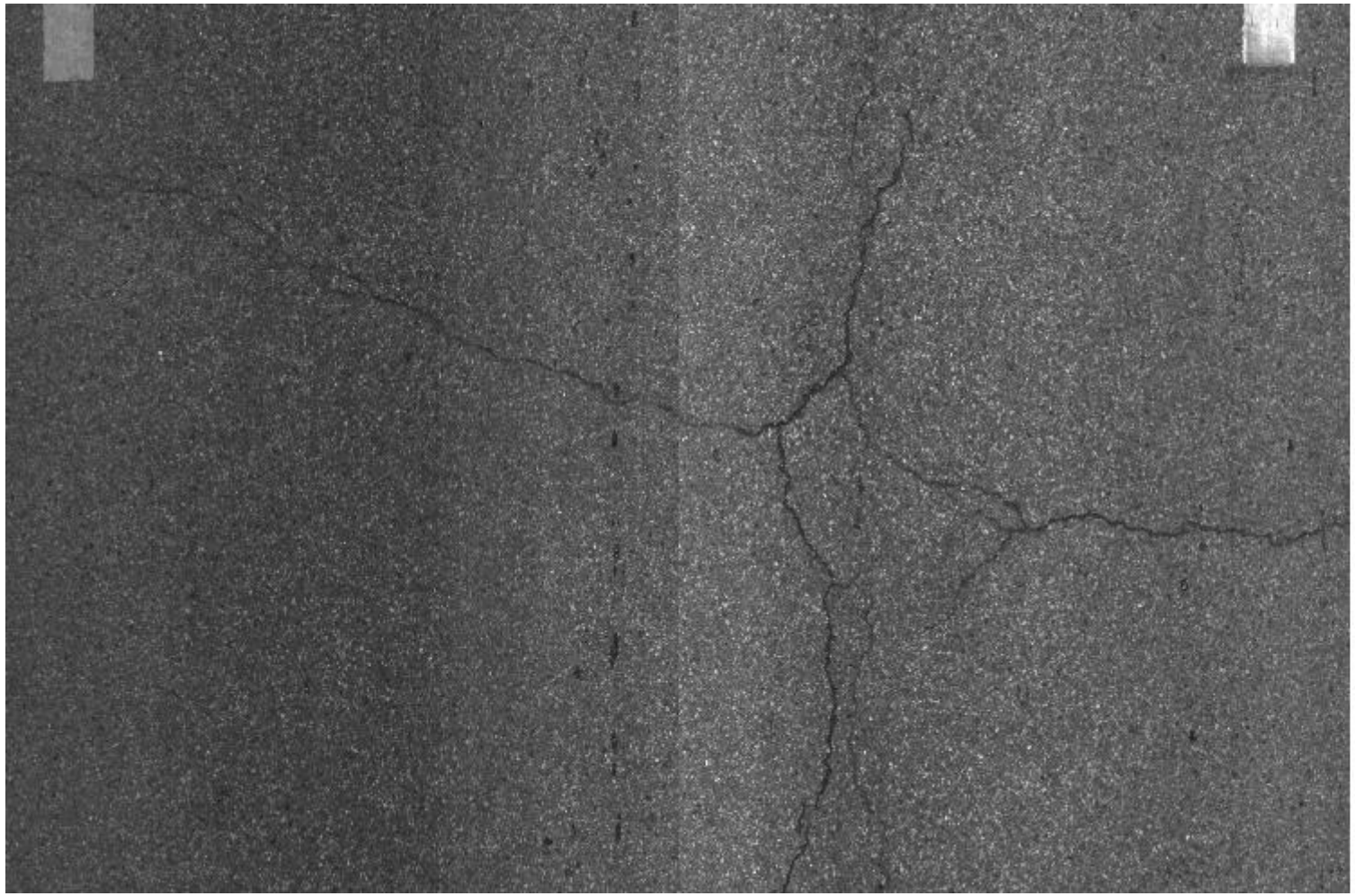

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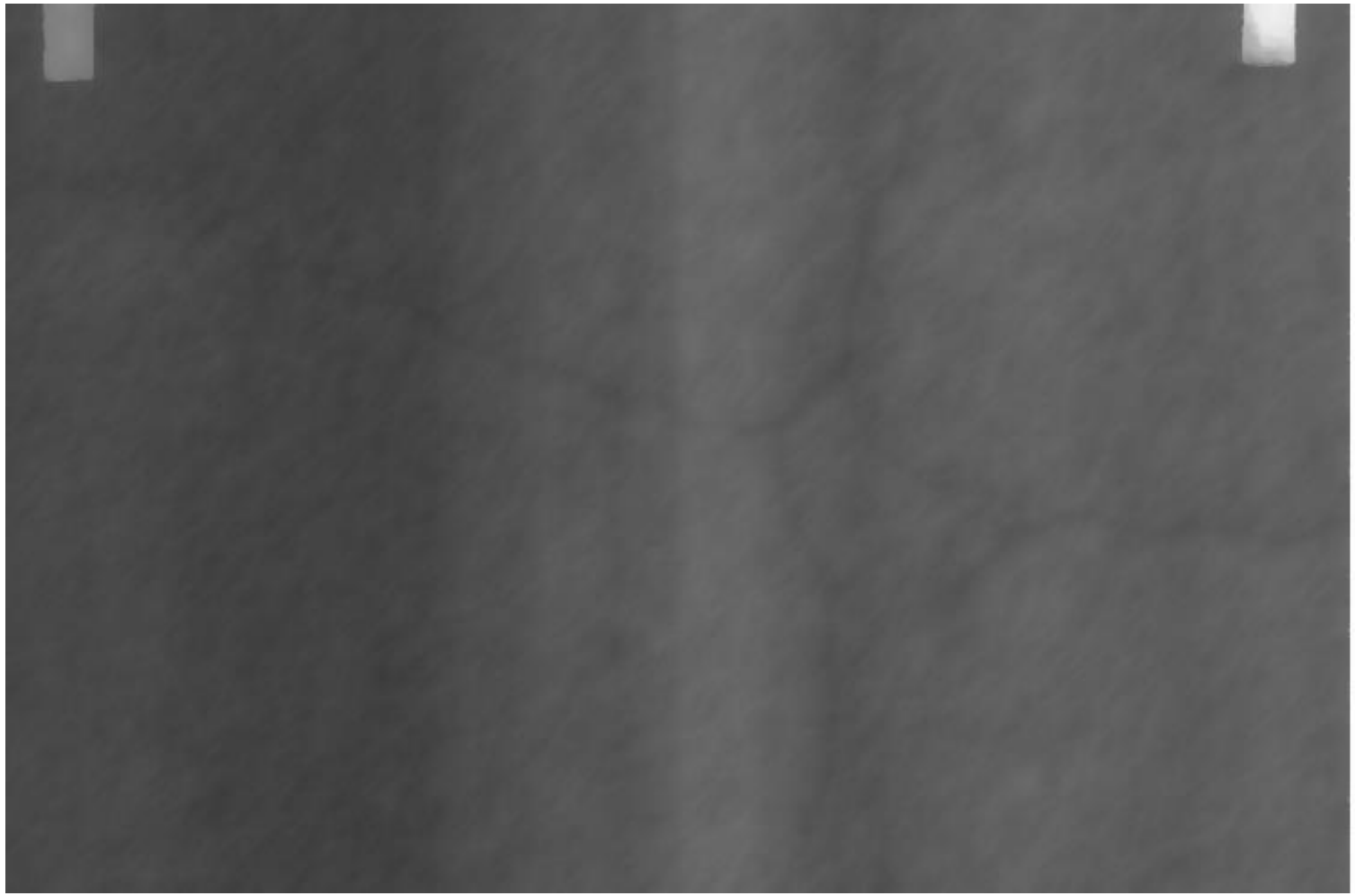
Image =

Archive de IPOL : http://demo.ipol.im/demo/103/archive



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Cartoon

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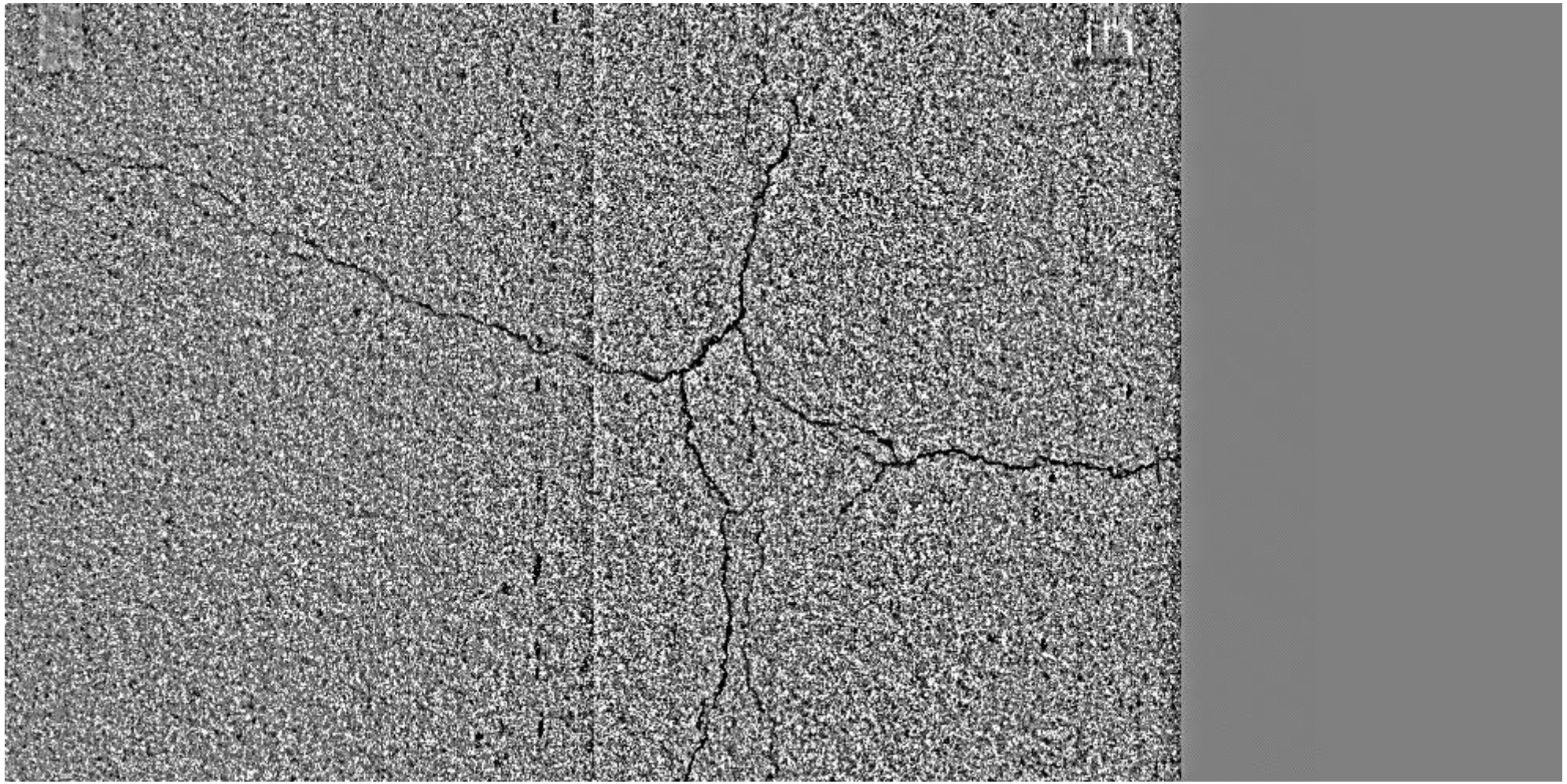
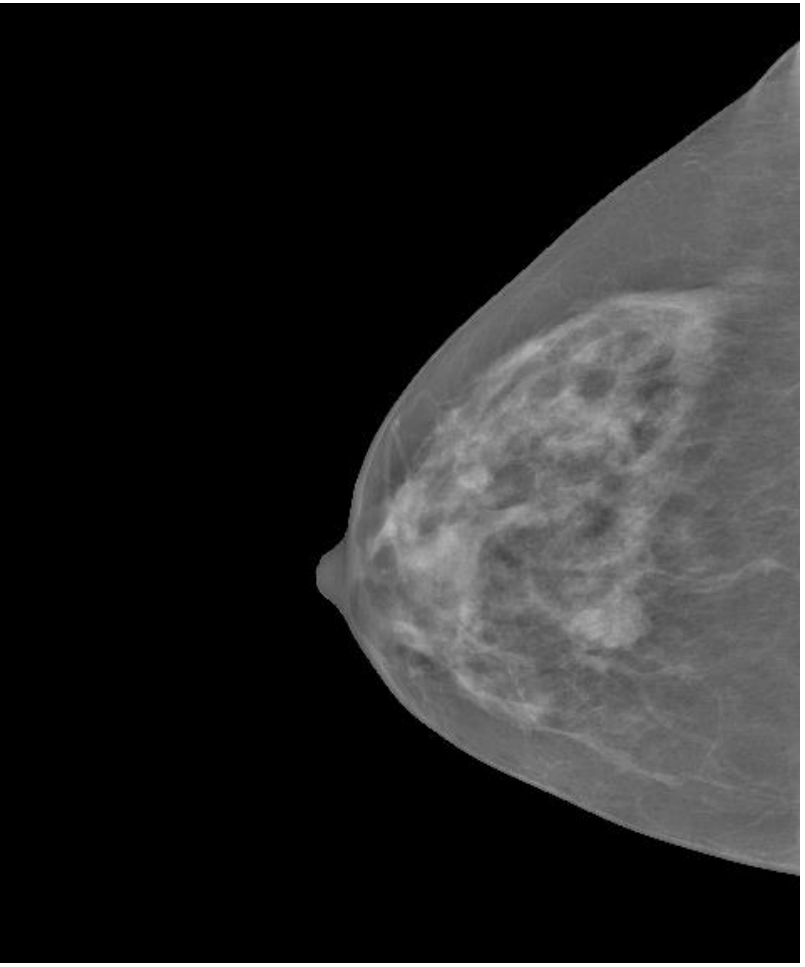
Archive de IPOL : http://demo.ipol.im/demo/103/archive



Texture (fissures bien plus faciles à détecter)



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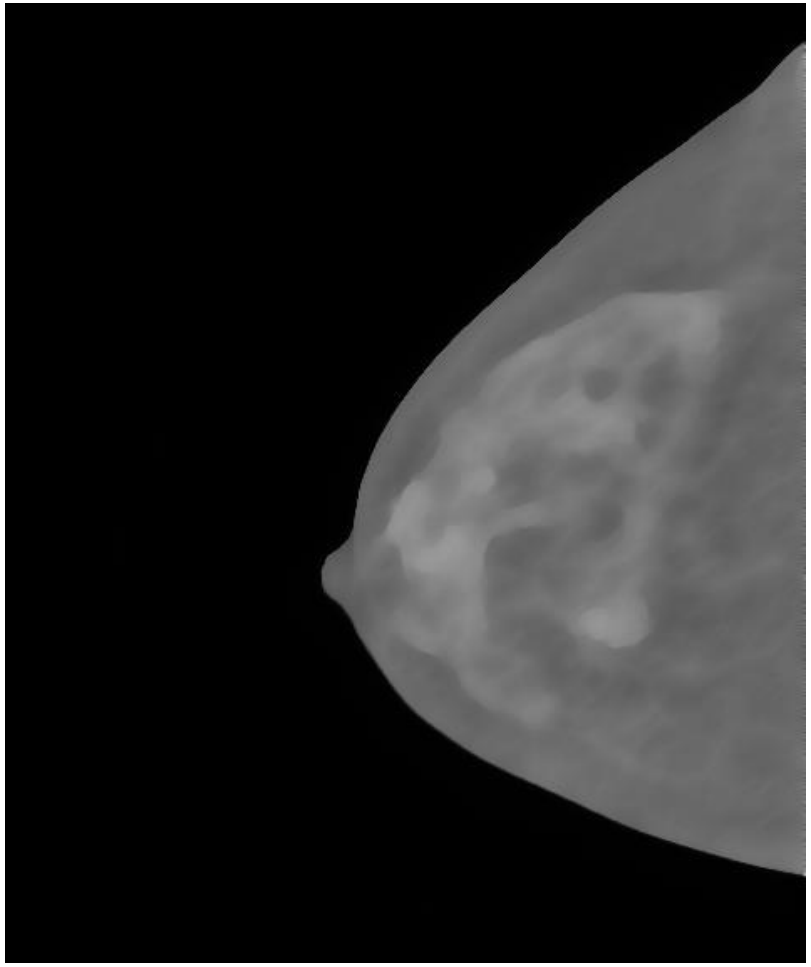

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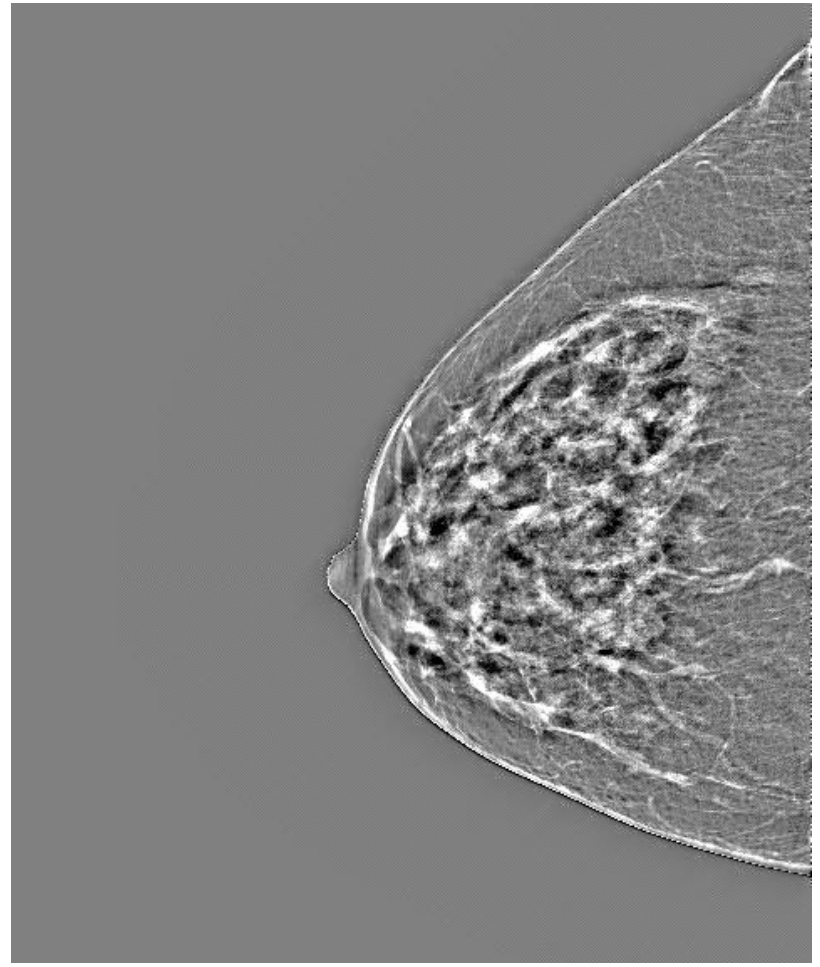

Image = Cartoon + Texture

Mammographie, détection de cancer du sein, rehaussement des détails dans la texture

Archive de IPOL : http://demo.ipol.im/demo/103/archive

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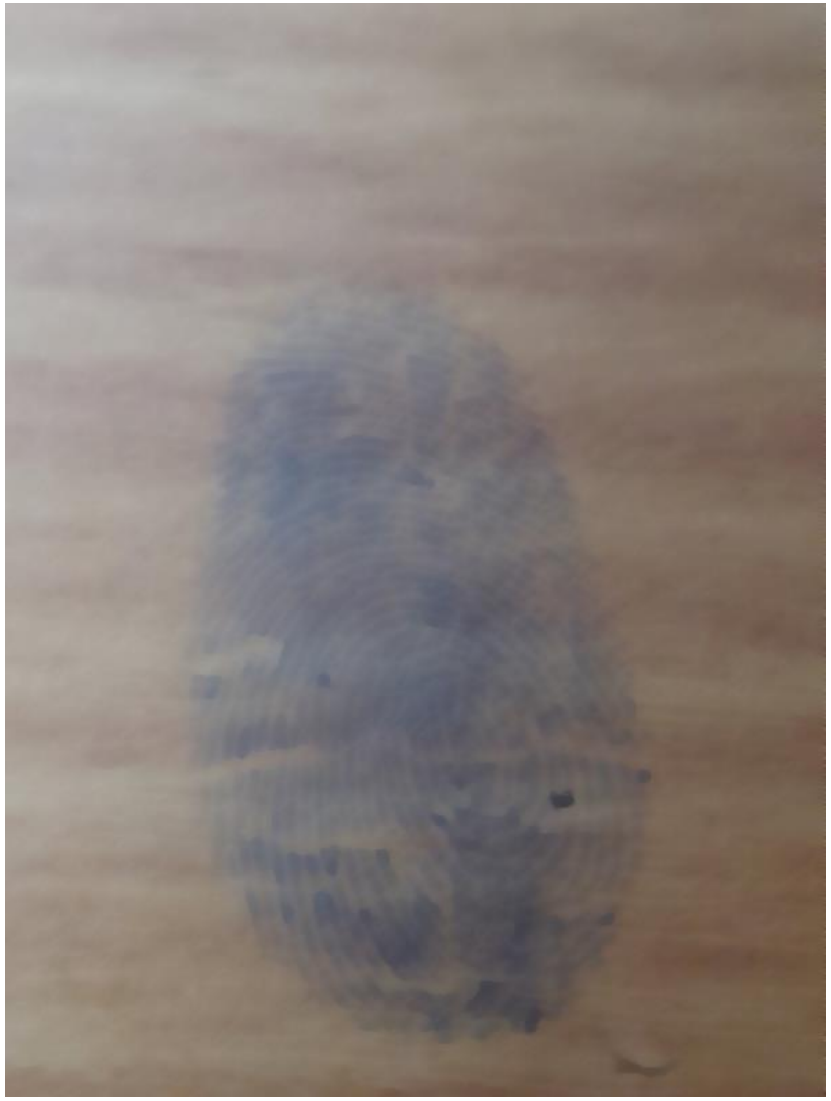

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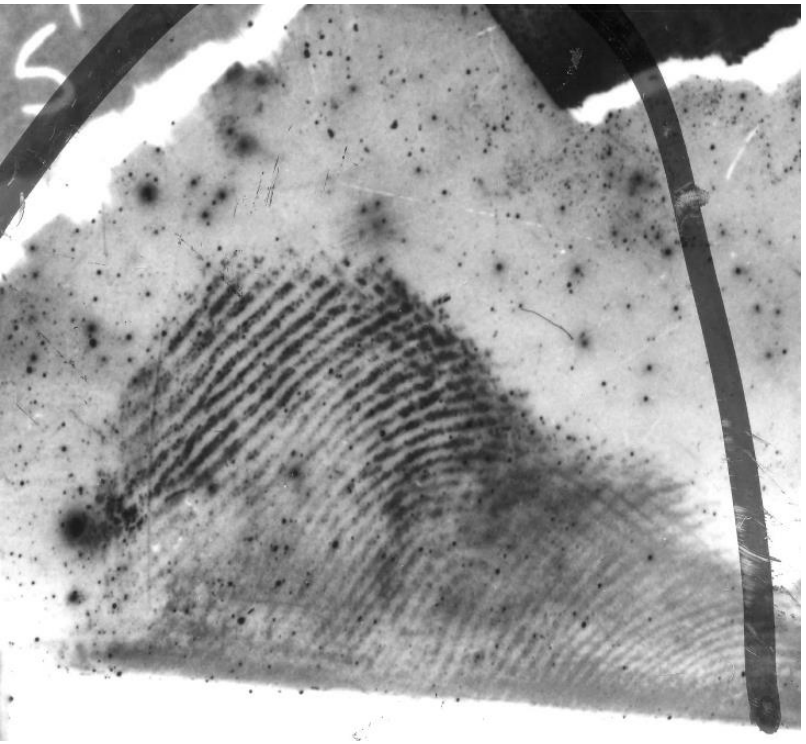
Image = Cartoon + Texture

Extraction d'empreintes digitales de (pour les soumettre à un système automatique de reconnaissance)

Archive de IPOL : http://demo.ipol.im/demo/103/archive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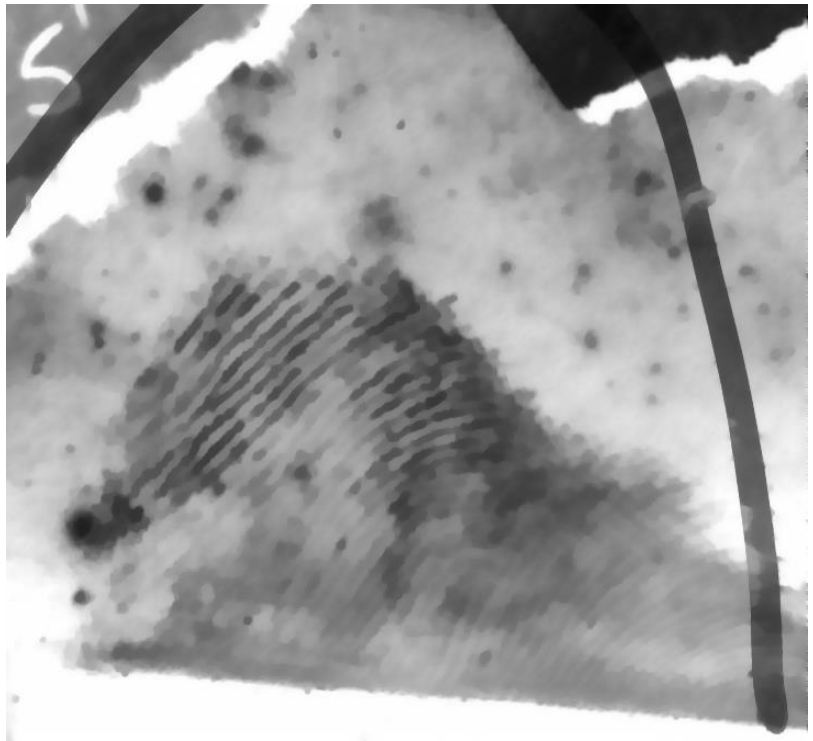
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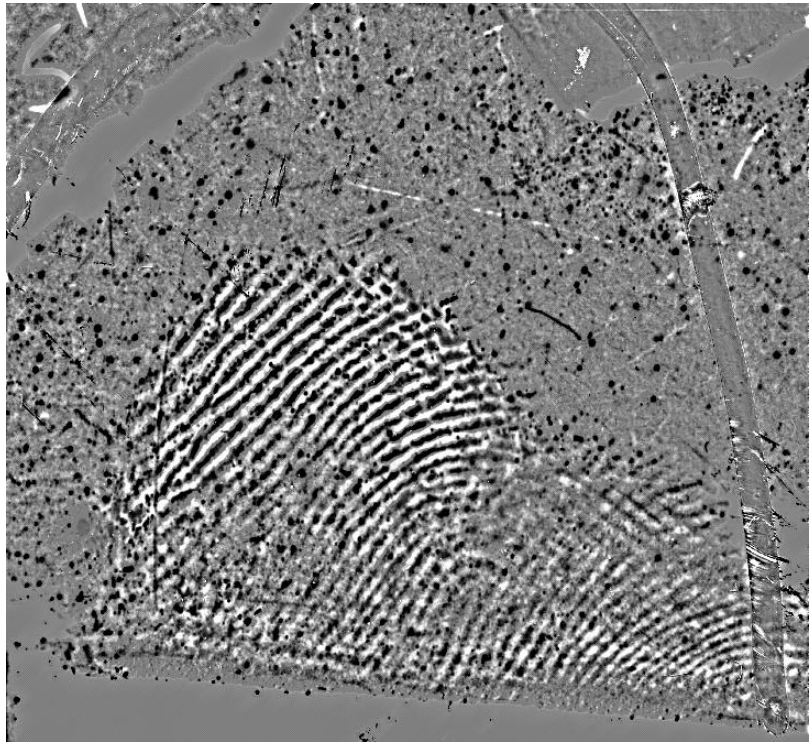

Image = Cartoon + Texture

Extraction d'empreintes digitales de (pour les soumettre à un système automatique de reconnaissance)

Archive de IPOL : http://demo.ipol.im/demo/103/archive

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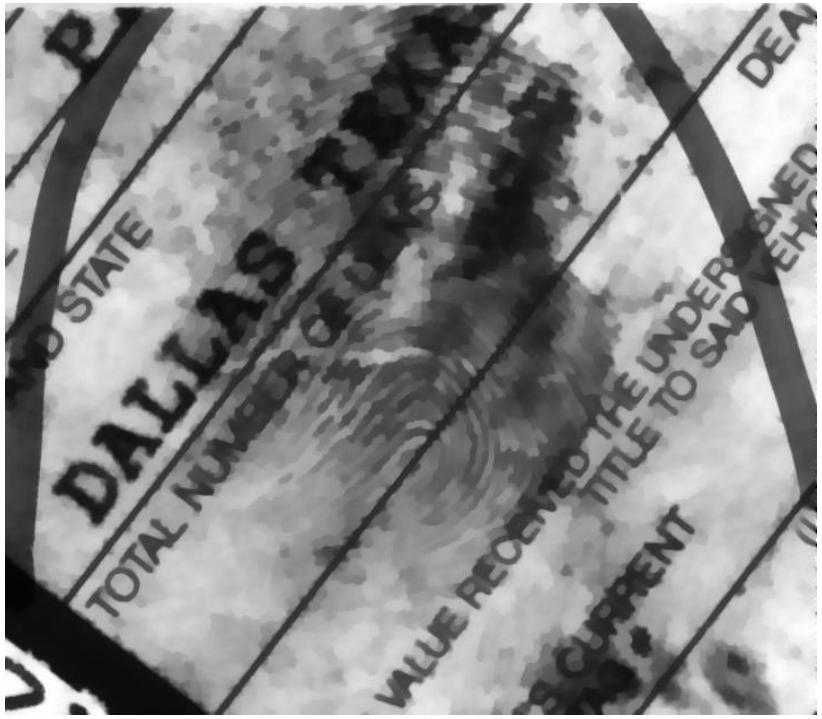

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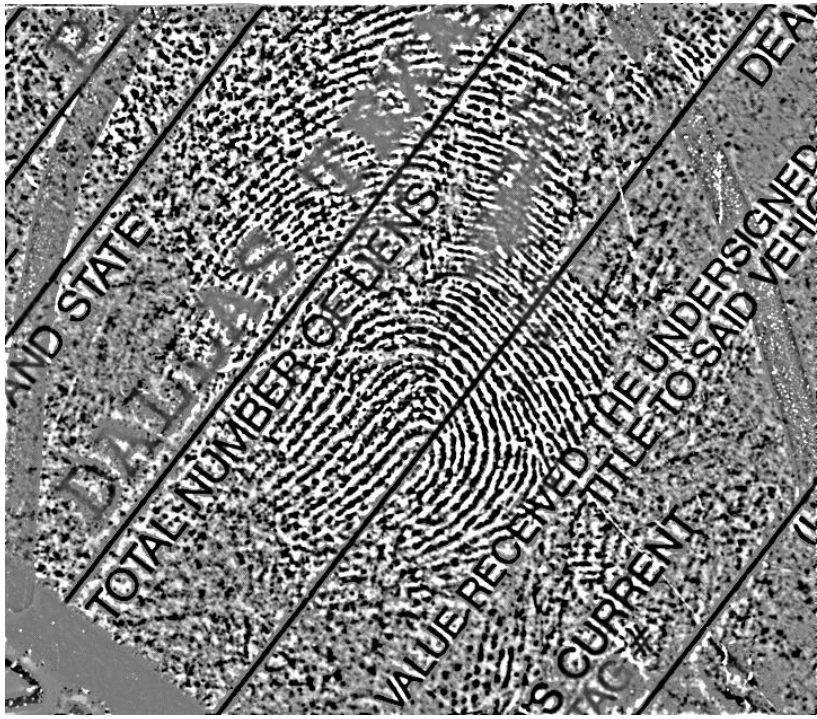

Image = Cartoon + Texture

Extraction d'empreintes digitales de (pour les soumettre à un système automatique de reconnaissance)

Archive de IPOL : http://demo.ipol.im/demo/103/archive

Image f =

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Archive de IPOL : http://demo.ipol.im/demo/103/archive

Cartoon u +

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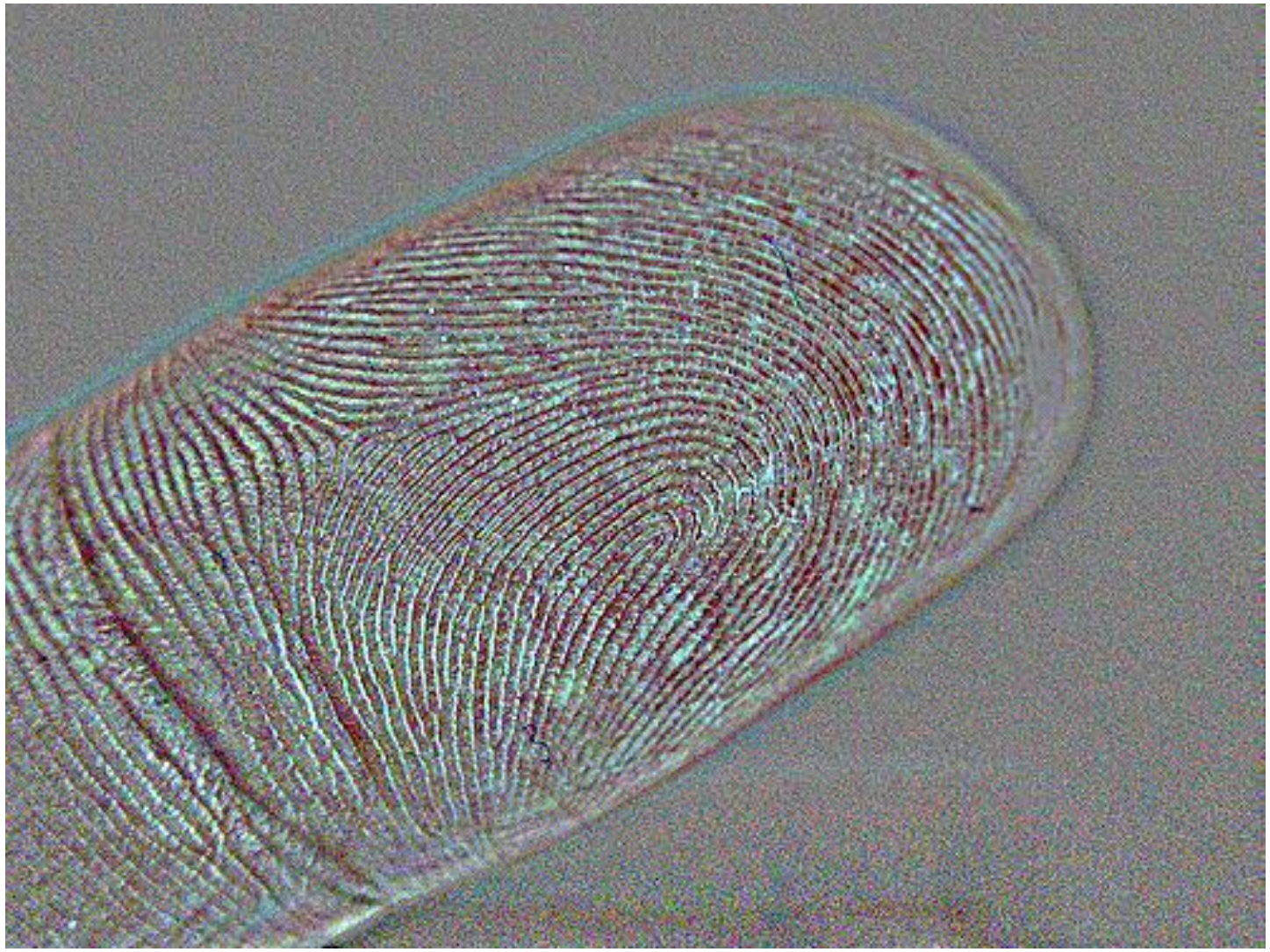
Archive de IPOL : http://demo.ipol.im/demo/103/archive



Texture v

Archive de IPOL : http://demo.ipol.im/demo/103/archive

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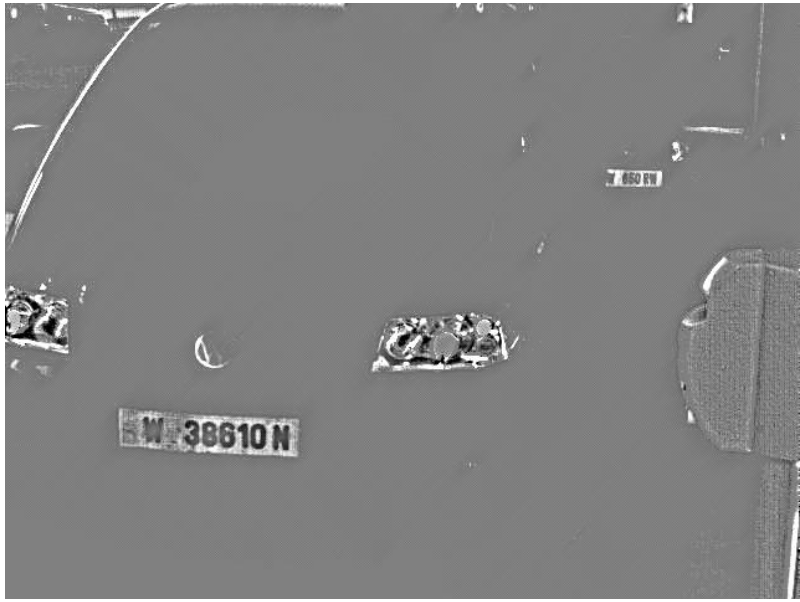

Image = Cartoon + Texture

Extraction de plaques d'immatriculation (pour reconnaissance automatique de caractères)

Archive de IPOL : http://demo.ipol.im/demo/103/archive

Image = Cartoon + Texture

Extraction de plaques d'immatriculation (pour reconnaissance automatique de caractères)

Archive de IPOL : http://demo.ipol.im/demo/103/archive

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Image = Cartoon + Texture

Extraction d'empreintes digitales de (pour les soumettre à un système automatique de reconnaissance)

Archive de IPOL : http://demo.ipol.im/demo/103/archive

Merci !

Retrouvez toutes ces expériences dans IPOL (Image Processing on Line) et faites en d'autres:
http://www.ipol.im/pub/art/2014/103/

(1) Yves Meyer *Oscillating patterns in image processing and nonlinear evolution equations (*2001)

(2) Leonid I. Rudin, Stanley Osher, and Emad Fatemi *Nonlinear total variation based noise removal algorithms* (1992)

(3) Vincent Le Guen *Cartoon + Texture Image Decomposition by the TV-L1 Model*, (2014)

(4) Jean-Francois Aujol, Guy Gilboa, Tony Chan & Stanley Osher, *Structure-Texture Image Decomposition* (2005)

(5) M. Nikolova *A variational approach to remove outliers and impulse noise*, 2004.

(6) Antonin Chambolle *An algorithm for total variation minimization and applications* (2004)

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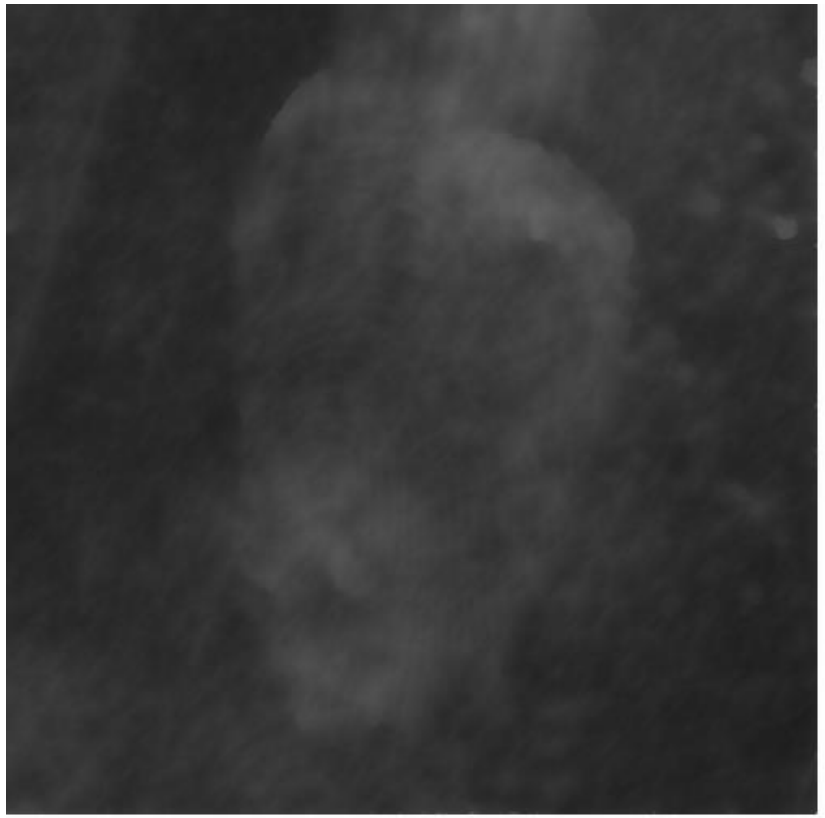

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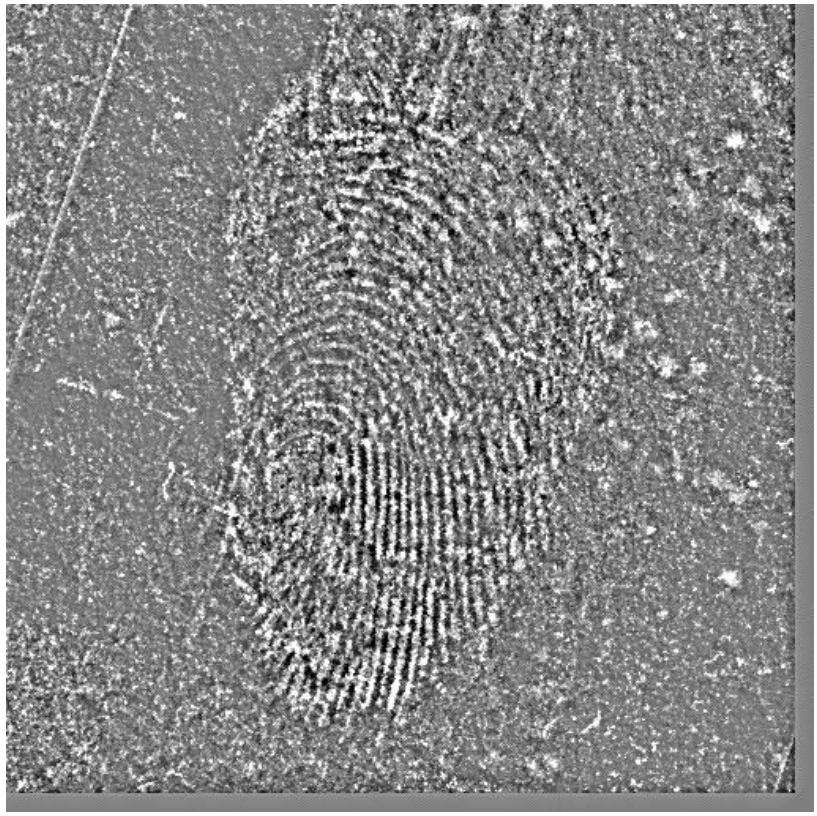

Image = Cartoon + Texture

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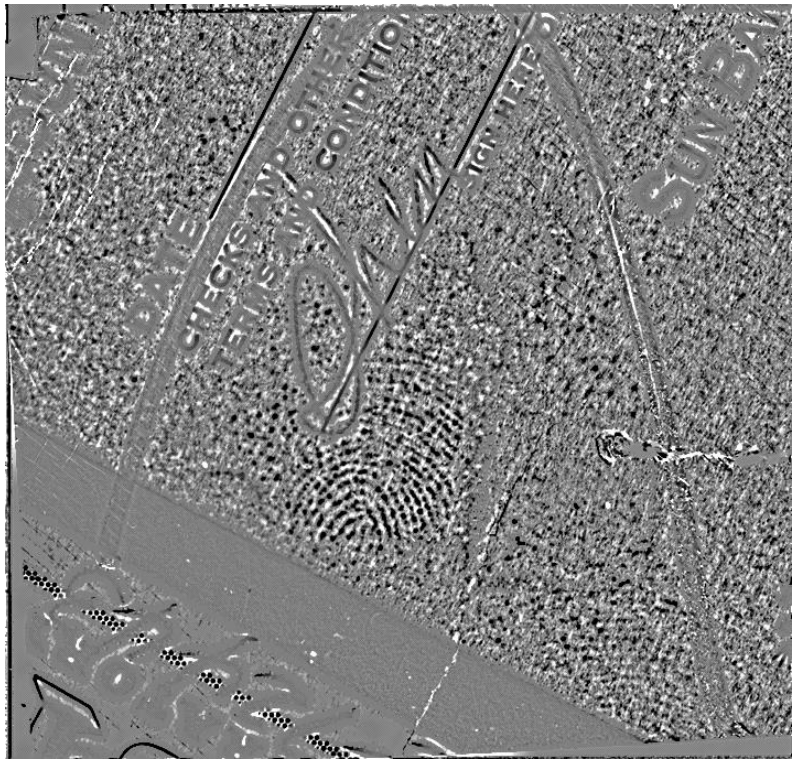

Image = Cartoon + Texture

Extraction d'empreintes digitales de (pour les soumettre à un système automatique de reconnaissance)

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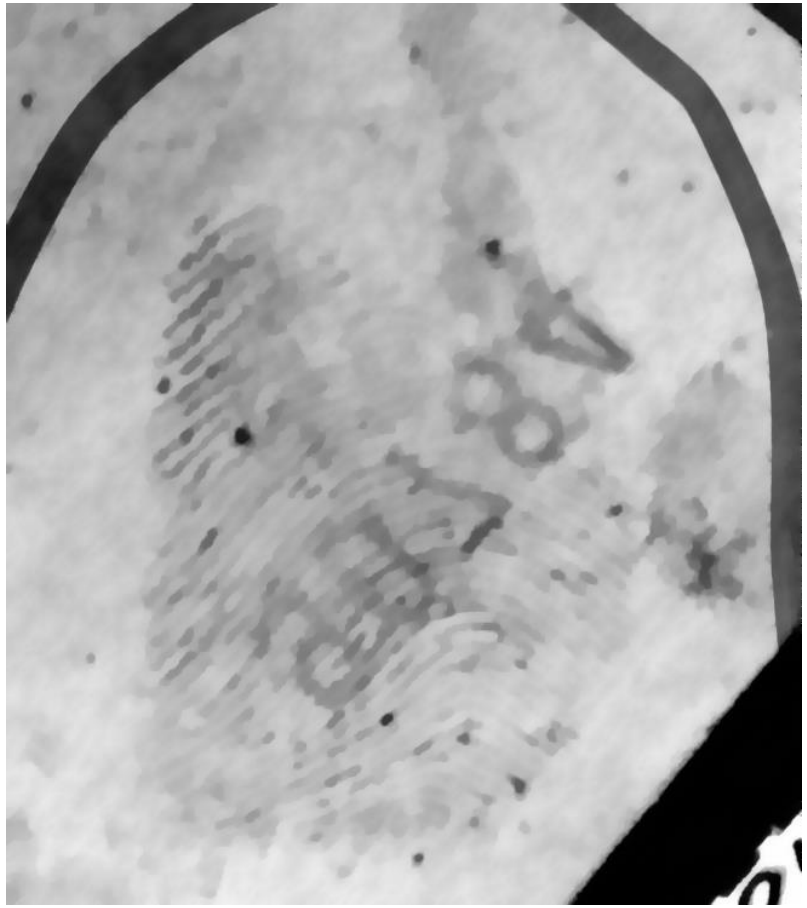

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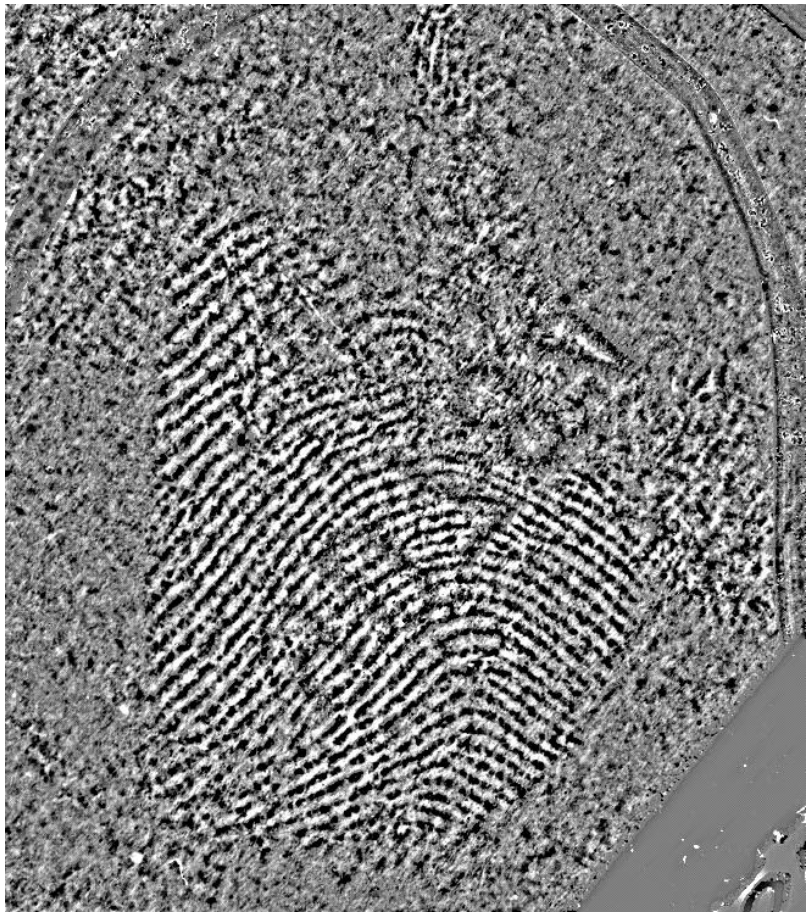

Image = Cartoon + Texture

Extraction d’empreintes digitales de (pour les soumettre à un système automatique de reconnaissance)

Image = Cartoon + Texture

Image = Cartoon + Texture

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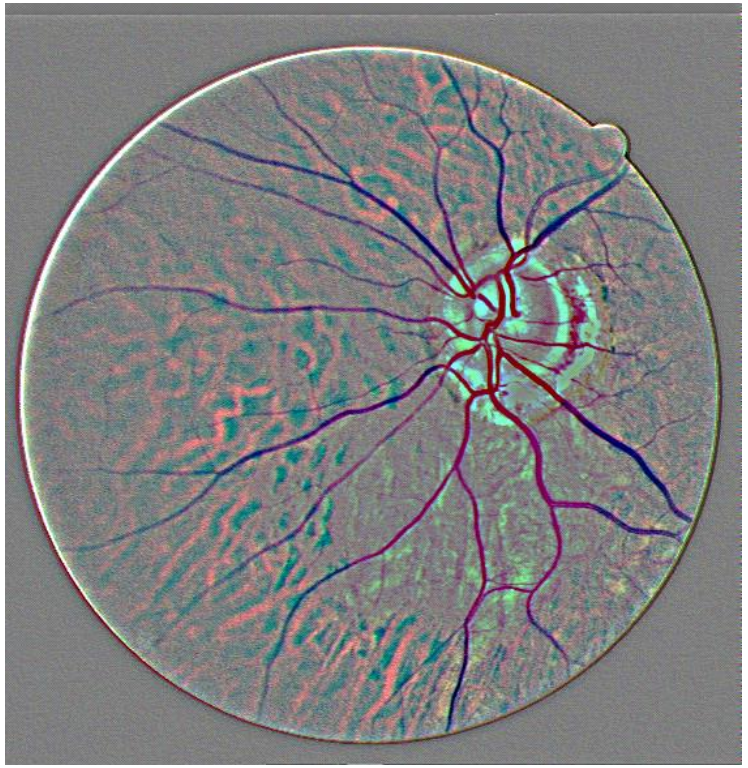

Image = Cartoon + Texture

Rétinoscopie (extraction du réseau sanguin)

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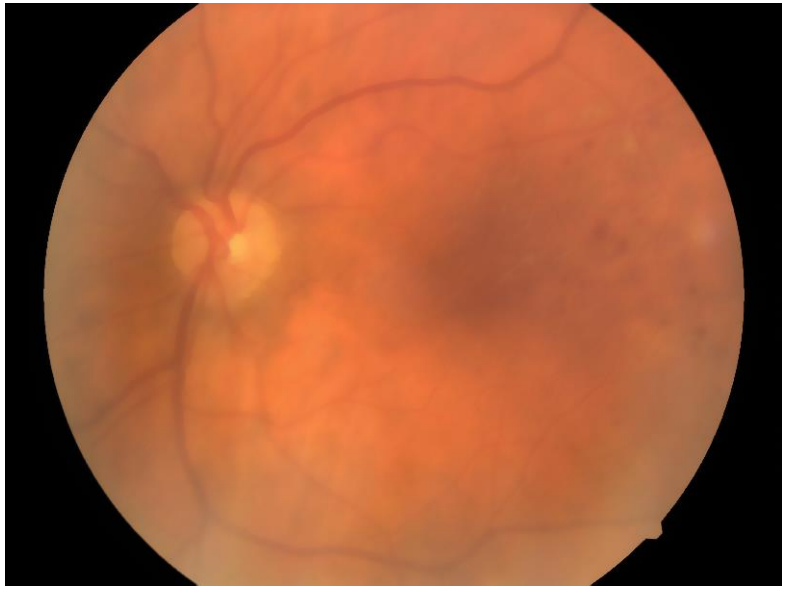

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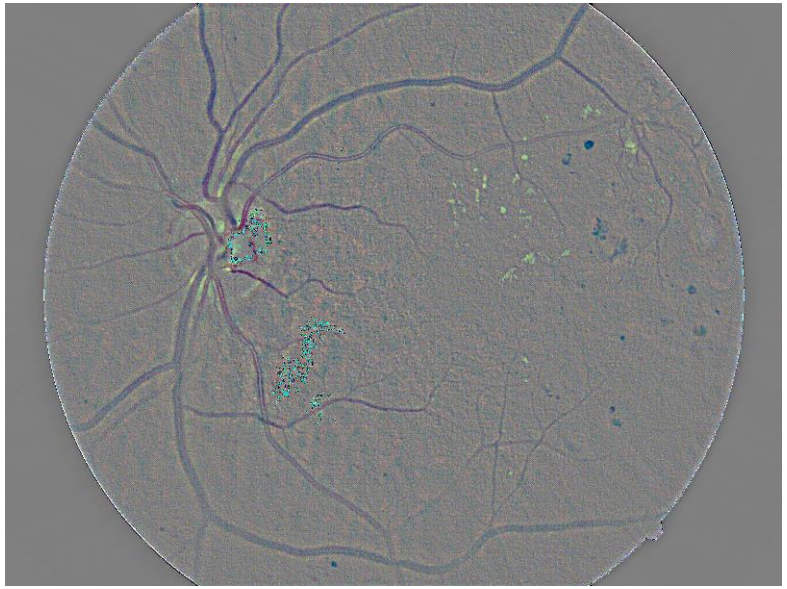

Image = Cartoon + Texture