Onsager lecture
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Sampling belongs to the toolbox of the digital revolution. Sampling a signal f on a grid $S = \{\ldots, s_{-1}, s_0, s_1, \ldots\}$ yields a sequence $X = (f(s_j))_{j\in\mathbb{Z}}$ of real numbers. The continuous world of signals or images is mapped into the discrete world of sequences. Is it possible to recover f from X ? Shannon's theorem answers this issue when f is a band limited signal and S is a regular grid ($s_j = jh$ where $h > 0$ is the step size). For a long time an irregular grid ($S \neq h\,\mathbb{Z}$) was viewed as a wrong choice while sampling on a regular grid was considered as a good fortune. To our greatest surprise the opposite is true. Sampling on a simple quasi-crystal improves on Shannon's theorem. This line of research is an illustration of the new paradigm of compressed sensing of sparse signals.

In the second part of this talk some new and still unpublished material will be presented. Sampling on general quasi-crystals raises several exciting issues. These problems will be addressed in the context of L^p norms ($1 \leq p \leq \infty$). This line of research is surprisingly connected to some previous work on mean-periodic functions.