Localized states and oscillations induced by coherent interaction of waves in nonlocal media

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Abstract: We show the dynamical four-wave mixing in a nonlocal medium is described by the complex Ginzburg-Landau equation. Two regimes are considered for this FWM: (i) the self-oscillations and (ii) stationary localized states in the form of sech-function for the intensity pattern in the medium volume. The applications of such FWM in all-optical switching and optical phase conjugation are examined.

I. INTRODUCTION

The property of spatial localization of the grating amplitude profile formed during dynamical four-wave mixing (FWM) in photorefractive media with nonlocal response (Fig.1) has been noted in several papers [1-4]. The damped sine-Gordon equation was obtained to describe the dynamics of the FWM in a nonlocal medium [4-6]. A regime of self-oscillations was found and investigated theoretically for such scheme [6, 7]. The first experimental observation of the spatial localization of the grating amplitude was provided in [7].

In the present paper we show that the damped sine-Gordon equation can be transformed to the cubic complex Ginzburg-Landau equation, which describes the properties of the dynamical FWM. We investigate the conditions required to get the self-oscillation regime. Also we investigate the properties of spatial localized states both for the intensity pattern and for the grating amplitude distribution that are described by a sech-function along the medium volume (the axis z). We show that the stationary distribution is controlled by the input intensity ratio. This fact gives methods to optimized different applications of the FWM. We consider here all-optical switching scheme and optical phase conjugation.

II. COMPLEX GINZBURG-LANDAU EQUATION DISCRIBED DYNAMICAL FWM IN NONLOCAL MEDIA

Transmission dynamical FWM is described by the set of four coupled-wave equations [7, 8]

$$\partial_z A_1 = -i\mathrm{E} A_2 \,,\, \partial_z \overline{A}_2 = i\mathrm{E}\overline{A}_1 \,,\, \partial_z \overline{A}_3 = -i\mathrm{E}\overline{A}_4 \,,\, \partial_z A_4 = i\mathrm{E} A_3 \quad (1)$$

and one evolution equation for the grating amplitude

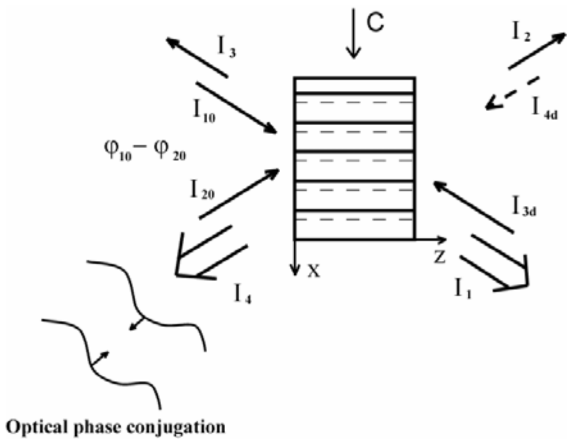


Fig. 1. The conventional scheme of the degenerate FWM in the transmission geometry. Straight lines are the maximums of the interference pattern, dash lines are the maximums of the grating amplitude. C is the polar axis. I_{10}, I_{20}, I_{3d}, I_{4d} are the input waves, I_1, I_2, I_3, I_4 are the output waves. The usual assumption is I_{4d}=0.

$$\partial_t \mathrm{E} = \gamma I_m / I_0 - \mathrm{E}/\tau \quad (2)$$

where the intensity pattern is $I_m = A_1\overline{A}_2 + \overline{A}_3 A_4$. Here $A_j(t,z),\ j = 1,2,3,4$ are the slow variable amplitudes of the interacted waves normalized to the total intensity $I_0 = |A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2 = const$; $\mathrm{E}(t,z)$ is the amplitude of the dynamical grating; τ is the time relaxation constant of the grating; γ is the photorefractive gain normalized to the time constant τ. In a nonlocal medium γ is a complex value.

We consider here a particular case of a pure nonlocal response when $\gamma = i\gamma_N$ is the pure imaginary. Then the system (1)-(2) is reduced to the damped sine-Gordon equation [4-8]:

$$\partial_t \partial_z u + \partial_z u / \tau - K \sin(2u + \alpha) = 0 \quad (3)$$

where $|E| = \partial_z u$ and the both coefficients K and α are depended on t. This equation has a solution in a form of bright soliton [8]:

$$I_m = |\mathrm{E}|/(\gamma_N \tau) = -ie^{-i(t-t_0)/\tau} \cdot \tau K \sec h(2\tau K(z - z_0)) \quad (4)$$

The damped sine-Gordon equation (3) is transformed to the cubic complex Ginzburg-Landau equation [8]:

$$\begin{aligned} & i\frac{\partial\psi}{\partial\eta} + \frac{2}{q^3}\frac{\partial^2\psi}{\partial\zeta^2} + \frac{4}{q}|\psi|^2\psi \cdot e^{-2T_0/\tau - 2\Im(q)Z_0} \\ & = -i\frac{2\Im(q)}{|q|^2}\frac{k_2}{k_0}\psi \end{aligned} \quad (5)$$

where the gain/loss coefficient k_2 is time depended and the wave-vector q is a complex constant. This way various solutions of the complex Ginzburg-Landau equation [9] may find their practical realizations during dynamical FWM in nonlocal media.

III. Self-oscillations in media with strong response

Stable oscillations (see Fig.2) are appeared in some area of input parameters due to influence of white noise to the phases of interacted waves:

$$\Phi(0) = \varphi_{10} - \varphi_{20} = \Phi_0 + \Lambda f(t), \Lambda \sim \pi/3 \div \pi/100 \quad (6)$$

where Λ is a scalar, $f(t)$ is a white noise. The amplitude and the period of the oscillations depend on the input conditions: the value of the coupling constant and the intensity ratio. The reason of the oscillation behaviour is the emergence of a local component of the grating that causes the changes of wave phases during their propagation; the light contrast changes with time, and the grating is erased and rerecorded repeatedly.

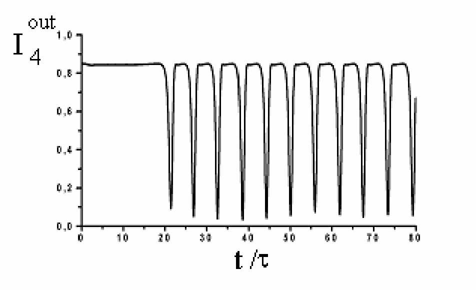


Fig. 2. The self-oscillations of the output intensity. I_{10}/I_{20}=3, I_{3d}=0.87, I_{4d}=0, $\gamma_N d$=15.

IV. Control of optical beams

In the steady state the solution for the intensity pattern and the grating amplitude distribution has a form of the sech-function [4-5, 7]:

$$\begin{aligned} I_m = \mathrm{E}/\gamma_N = C/\cosh(2\gamma_N Cz - p), \\ tg(u) = \exp(2\gamma_N Cz - p) \end{aligned} \quad (7)$$

where C and p are the constants defined by the input conditions of a FWM scheme. The main parameter is u_d, which is the integral under the grating amplitude profile over the medium boundaries. The value u_d determines the diffraction efficiency:

$$u_d = \int_0^d \mathrm{E}(z)dz, \quad (8)$$

for the two-wave mixing:

$$\eta = \sin^2[\gamma_N u_d + arctg(A_{10}/A_{20})]$$

The value u_d depends on the location of the stationary sech-function of $\mathrm{E}(z)$ relative to the crystal boundaries $z = 0 \div d$. To characterize FWM schemes with different input conditions we introduce new parameters: the position of the grating amplitude maximum Z_0 and the half-width of the grating amplitude distribution Δ:

$$Z_0 = p/2\gamma_N C, \quad \Delta = \ln(3+\sqrt{2})/\gamma_N C \quad (9)$$

The calculations of the value u_d in dependence of these parameters for the FWM schemes with two different input conditions are combined in Fig.3. However only a part of the presented curves will correspond to any concrete scheme. This way one can choose optimal input conditions for different applications.

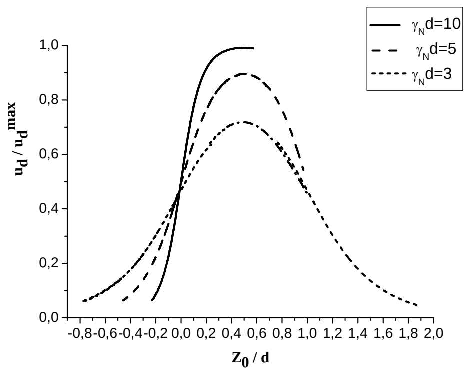
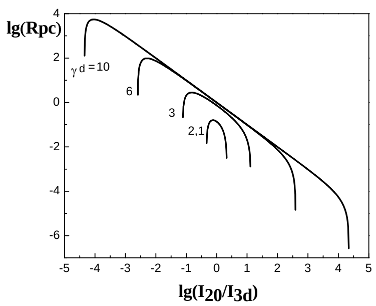


Fig. 3. The normalized value u_d calculated for the FWM with two driven beams when their input intensity ratio is changed on the 3 orders of magnitude.

Optical switching. As it is following from the Fig.3 the optical switching is realized if to change u_d from its maximum value to its minimum value with the help of a guided beam. This situation occurs on the base of the transmission two-wave mixing scheme, where input beams I_1 and I_2 record the grating. But the third beam I_{3d} is the guided one, which switch on/off the value of the u_d and by this way the diffraction efficiency (see Fig.4).

Optical phase conjugation (OPC). OPC is realized when $u_d = u_{d\max}$ over all range of changes of the input intensities. The optical scheme is on the base of the double phase conjugation mirror (DPCM) when $I_{10} = I_{4d} = 0$. The DPCM scheme is unstable. But introducing a weak beam $I_{10} \neq 0$ makes the stabilization of this scheme (see Fig.5 a,b).

(a)



(b)

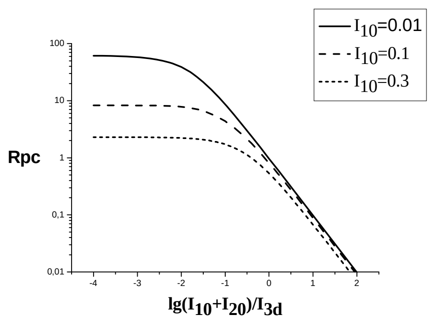


Fig. 5. The coefficient of the phase conjugation Rpc=I4(0)/I_{20} in DPCM (a) and in the FWM scheme based on the DPCM scheme (b) - $\gamma_N d=6$, $I_{10}+I_{20}+I_{4d}=1$.

V. CONCLUSION

We show rigorously that a nonlinear system describing the degenerate wave mixing in a medium which possesses both a nonlocal response and relaxation is reduced to one nonlinear complex Ginzburg-Landau equation (CGLE). We develop the technique to obtain the cubic CGLE by using the reductive perturbation method for the nonlinear dynamical system described wave coupling of four waves.

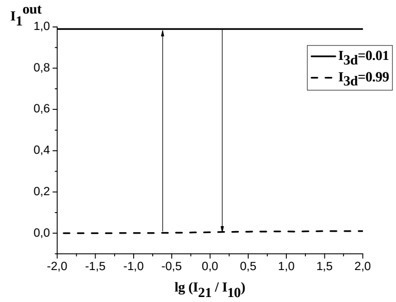


Fig. 4. All-optical switching on the base of two-wave mixing scheme. $I_{10}+I_{20}+I_{3d}=1$, $I_{4d}=0$, $\gamma_N d=10$.

The CGLE governs the spatiotemporal dynamics of the spatially localized interference pattern formed by the FWM. These properties of degenerate FWM in nonlocal media may find numerous applications, e.g. for optical phase conjugation, all-optical switching, manipulation of laser pulses, optical logic, transmission of solitary waves through fibers etc.

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